

Scientific AI for Cosmology and Beyond

Jason D. McEwen

www.jasonmcewen.org

Scientific AI (SciAI) Group, Mullard Space Science Laboratory (MSSL) University College London (UCL)

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Towards a fundamental understanding of our Universe



- ▷ Unanswered fundamental questions
- ⊳ Imminent new data
- ▶ How can we bring AI to bear?



1. Towards scientific AI

2. Statistical characterisation and generative modelling of cosmological fields



Towards scientific AI

The AI hammer









Merging paradigms





Scientific AI for the physical sciences





Physics Enhanced Learning

Embed physical understanding of the world into machine learning models.

(See review by Karniadakis et al. 2021.)









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Apply **physical transformations** that data known to satisfy to augment training data \rightsquigarrow ML model **learns physics through training**.

 Common to augment image data-set with rotations, flips, shifts, scales, contrast, ...



Image augmentation



Apply **physical transformations** that data known to satisfy to augment training data \rightsquigarrow ML model **learns physics through training**.

 Redshift augmentation of supernovae observations (Boone 2019; Alves, *et al.* 2022, 2023)



Redshift augmentation





Apply **physical transformations** that data known to satisfy to augment training data \rightsquigarrow ML model **learns physics through training**.



▷ Data efficiency suffers: data "used" to learn physics, rather than problem.



▷ Simple and easy to implement.



Encode physical properties of the world into ML models (e.g. geometry, symmetries, conservation laws) ~> **Physics embedded in architecture** of ML model.



Encode physical properties of the world into ML models (e.g. geometry, symmetries, conservation laws) → Physics embedded in architecture of ML model.

▷ Key factor CNNs so successful is due to encoding translational equivariance.





Translational equivariance

O Encode physical properties of the world into ML models (e.g. geometry, symmetries, conservation laws) → Physics embedded in architecture of ML model.

 Geometric deep learning on the sphere (Cobb et al. 2021; McEwen et al. 2022; Ocampo, Price & McEwen 2023)



CMB observed on the celestial sphere



O Encode physical properties of the world into ML models (e.g. geometry, symmetries, conservation laws) → Physics embedded in architecture of ML model.

 Equivariant machine learning, structured like classical physics (Villar et al. 2021) $\begin{array}{ll} & \text{Orthogonal} & \text{O}(d) = \{Q \in \mathbb{R}^{d \times d} : Q^\top Q = Q \, Q^\top = I_d\},\\ & \text{Rotation} & \text{SO}(d) = \{Q \in \mathbb{R}^{d \times d} : Q^\top Q = Q \, Q^\top = I_d, \det(Q) = 1\}\\ & \text{Translation} & \text{T}(d) = \{w \in \mathbb{R}^d\}\\ & \text{Euclidean} & \text{E}(d) = \text{T}(d) \times \text{O}(d)\\ & \text{Lorentz} & \text{O}(1, d) = \{Q \in \mathbb{R}^{(d+1) \times (d+1)} : Q^\top \Lambda \, Q = \Lambda, \Lambda = \text{diag}([1, -1, \ldots, -1])\}\\ & \text{Poincaré} & \text{IO}(1, d) = \text{T}(d + 1) \times \text{O}(1, d)\\ & \text{Permutation} & \mathbf{S}_n = \{\sigma : |n| \to |n| \text{ bijcetive function}\} \end{array}$

Groups considered



O Encode physical properties of the world into ML models (e.g. geometry, symmetries, conservation laws) → Physics embedded in architecture of ML model.

- ▷ Inductive biases required? Should we just learn from data?
 - ▷ Highly computationally demanding.

▷ Improved data-efficiency.



▷ Inductive biases not necessarily strictly enforced.



▷ Develop efficient algorithms (e.g. Ocampo, Price & McEwen 2023).

Encode physical models of world into ML models:

- 1. Encode dynamics (differential equations) via loss functions (PINNs).
- 2. Embed full (differentiable) physical models inside ML model.
- \rightsquigarrow Physics learned in training and embedded in model.



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Encode physical models of world into ML models:

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- ~ Physics learned in training and embedded in model.
- Physics informed neural networks (PINNs) encode differentiable equations (e.g. boundary conditions) in loss.



PINNs



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- 2. Embed full (differentiable) physical models inside ML model.
- ~ Physics learned in training and embedded in model.
- ▷ Differentiable physical models
 - Radio interferometric telescope (Mars et al. 2023, 2024, Liaudat et al. McEwen 2024)
 - Optical PSF

(Liaudat et al. 2023)

► JAX-Cosmo (Campagne et al. 2023)



SKA (artist impression)



(j)

Encode physical models of world into ML models:

- 1. Encode dynamics (differential equations) via loss functions (PINNs).
- 2. Embed full (differentiable) physical models inside ML model.
- ~ Physics learned in training and embedded in model.
- Differentiable mathematical methods
 - ▶ Fourier transforms
 - Spherical harmonic transforms (s2fft; Price & McEwen 2023)
 - Spherical wavelet transforms (s2wav; Price et al. McEwen 2024)
 - ▶ Spherical scattering transforms



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(s2scat; Mousset et al. McEwen 2024)



Spherical harmonics

Encode physical models of world into ML models:

- 1. Encode dynamics (differential equations) via loss functions (PINNs).
- 2. Embed full (differentiable) physical models inside ML model.
- ~ Physics learned in training and embedded in model.
- ▷ PINNs only capture limited dynamics via loss.
- ▷ Full physical models requires differentiable programming frameworks.

- ▷ Capture full physics with differentiable models!
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▷ Emulators also provide differentiability (e.g. CosmoPower; Spurio Mancini et al. 2021).



▷ Write new differentiable codes (e.g. s2fft; Price & McEwen 2023).

Probabilistic Learning

Embed a probabilistic representation of data, models and/or outputs.

(See Murray 2022.)









MC Dropout (Gal & Ghahramani 2016): drop nodes probabilistically to sample an ensemble of networks.





 Bayes by Backprop (Blundel *et al.* 2015): model distribution of weights (by variational inference).





 Probabilistic ML frameworks (*e.g.* TensorFlow Probability).





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Bayesian neural networks incorporate **probabilistic representation** to quantify **uncertainty of outputs** (idea pioneered by MacKay 1992).

- ▷ Encode epistemic uncertainty of model.
- ▷ But what does the output distribution represent?
- ▷ Requires careful consideration of training data.



▷ Statistical validation (hold that thought... see upcoming Truthfulness section).



Emulation: sample from learned prior
 (Perraudin et al. 2020, Allys et al. 2020, Price et al.
 2023, Price et al. in prep., Mousset et al. McEwen
 2024)



Emulated cosmic string maps (stringgen, Price *et al.* 2023, Price *et al.* in prep.)



 Integrate learned priors into analysis
 (Remy et al. 2022, McEwen et al. 2023, Liaudat et al. McEwen 2024)



Learn radio galaxy prior (Liaudat *et al.* McEwen 2024)



- \triangle
- $\triangleright\,$ Availability and representativeness of training data.
- ▷ Truthfulness, *e.g.* diversity of ML model often lacking.



- ▷ Public datasets/benchmarks (e.g. IllustrisTNG, CAMELS, Quijote, CosmoGrid, Gower St).
- ▷ Meta sampling to recover distribution over manifold (*e.g.* Price *et al.* 2023).



▷ Truthfulness (hold that thought... see upcoming Truthfulness section).

Bayesian inference

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ML techniques can be integrated into Bayesian frameworks to **enhance accuracy and computational efficiency**, making some approaches accessible that were previously intractable.



Bayesian inference

ML techniques can be integrated into Bayesian frameworks to **enhance accuracy and computational efficiency**, making some approaches accessible that were previously intractable.

▷ Enhanced MCMC for parameter estimation (Grabrie *et al.* 2022, Karamanis *et al.* 2022).



Learned proposal distributions



Bayesian inference

ML techniques can be integrated into Bayesian frameworks to **enhance accuracy and computational efficiency**, making some approaches accessible that were previously intractable.

- Enhanced Bayesian model selection
 (harmonic; McEwen et al. 2021, Polanska et al. 2024, Piras et al. McEwen 2024, Spurio Mancini et al. McEwen 2023, 2024).
 - ▶ Only requires posterior samples.
 - ► Agnostic to sampling technique.
 - ▶ Scale to high dimensions.




Bayesian inference

ML techniques can be integrated into Bayesian frameworks to **enhance accuracy and computational efficiency**, making some approaches accessible that were previously intractable.

▷ Simulation-based inference (Cranmer *et al.* 2021).



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Bayesian inference

ML techniques can be integrated into Bayesian frameworks to **enhance accuracy and computational efficiency**, making some approaches accessible that were previously intractable.

▷ Variational inference (Whitney *et al.* McEwen 2024)



Mass mapping with uncertainties by variational inference (Whitney *et al.* McEwen 2024)



Bayesian inference

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ML techniques can be integrated into Bayesian frameworks to **enhance accuracy and computational efficiency**, making some approaches accessible that were previously intractable.



- ▷ Availability and representativeness of training data.
- ▷ Cost of training.
- ▷ Truthfulness?
- ▷ Public datasets/benchmarks (*e.g.* IllustrisTNG, CAMELS, Quijote, CosmoGrid, Gower St).



- ▷ Amortized inference (training **not** repeated for new observations).
- ▷ Integrate in Bayesian framework to provide statistical guarantees.



▷ Statistical validation (hold that thought... see upcoming Truthfulness section).

Intelligible AI

Machine learning methods that are able to be understood by humans.

(See Weld & Bansal 2018, Ras et al. 2020.)







Explainable ML techniques may or may not be interpretable themselves but their **outputs can be explained to humans.**



• Explainable ML techniques may or may not be interpretable themselves but their outputs can be explained to humans.

▷ Feature importances (Lochner *et al.* 2016)



Supernova feature importances



Explainable ML techniques may or may not be interpretable themselves but their outputs can be explained to humans.

Saliency maps
 (Bhambra *et al.* 2022)



Galaxy saliency mapping





Explainable ML techniques may or may not be interpretable themselves but their **outputs can be explained to humans.**



Poking the black box: may provide some explanation of outputs but humans still not able to comprehend underlying process.







 Designed models such as wavelet scattering networks
 (Allys *et al.* 2020, Cheng *et al.* 2020, McEwen *et al.* 2022, Mousset *et al.* McEwen 2024)



Scattering network (McEwen et al. 2022)



 Designed models such as wavelet scattering networks
 (Allys et al. 2020, Cheng et al. 2020, McEwen et al. 2022, Mousset et al. McEwen 2024)



LSS features captured by wavelets (Allys *et al.* 2020)



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Interpretable ML models are white boxes that can be understood by humans.

 Interpretable constraints on ML models, e.g. convexity (Liaudat et al. McEwen 2024)



Impose convexity on learned model



 Deep priors learned from training data (hybrid model-based and data-driven)
 (Remy et al. 2022, McEwen et al. 2023, Liaudat et al. McEwen 2024)



Compute Bayesian evidence for model selection (proxnest, McEwen *et al.* 2023)





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- ▷ Designed models limit flexibility.
- ▷ Availability and representativeness of training data.



- ▷ Benefits of designed models often outweigh (minimal) reduced flexibility.
- ▷ Public datasets/benchmarks (*e.g.* IllustrisTNG, CAMELS, Quijote, CosmoGrid, Gower St).





Truthfulness **critical for science** in order for humans to have confidence in results of ML models. Closely coupled with a **meaningful statistical distribution** of outputs.



D Truthfulness **critical for science** in order for humans to have confidence in results of ML models. Closely coupled with a **meaningful statistical distribution** of outputs.

- Validity of statistical distributions (Lueckmann *et al.* 2021, Hermans *et al.* 2022, Cannon *et al.* 2023)
 - ► Design to ensure conservative and avoid mode collapse (Delaunoy *et al.* 2022, Price *et al.* 2023, Whitney *et al.* McEwen 2024)
 - ▶ Coverage testing (Lemos *et al.* 2023)
 - ► Simulation-based calibration checks (Talts *et al.* 2020)



Validity of distribution (Hermans *et al.* 2022)





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Truthfulness **critical for science** in order for humans to have confidence in results of ML models. Closely coupled with a **meaningful statistical distribution** of outputs.

- ▷ Uncertainties not aways meaningful.
- ▷ Diversity of ML model often lacking.

▷ Integrate in statistical framework to inherit theoretical guarantees.



- $\triangleright\,$ Design to be conservative and avoid mode collapse.
- ▷ Extensive validation tests.



▷ Well-posed frameworks (*e.g.* physics enhanced, probabilistic).

Statistical characterisation and generative modelling of cosmological fields

Wavelet scattering networks and representations inspired by CNNs but designed rather than learned filters (Mallat 2012).

→ Scattering networks on the sphere (McEwen et al. 2022, ICLR, arXiv:2102.02828)

→ Generative models of astrophysical fields with scattering transforms on the sphere (Mousset *et al.* McEwen 2024, A&A, arXiv:2407.07007)



Wavelets on the sphere

Adopt scale-discretized wavelets on the sphere (e.g. McEwen et al. 2018, McEwen et al. 2015).

Wavelets $\psi_j \in L^2(\mathbb{S}^2)$ capture spatially-localised, high-frequency signal content at scale *j*.

Scaling function $\phi \in L^2(\mathbb{S}^2)$ captures spatially-localised, low-frequency content.



Orthographic plot of spherical wavelets.



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Spherical wavelet transform given by

$$W_{j}(\rho) = (f \star \psi_{j})(\rho) = \int_{\mathbb{S}^{2}} d\mu(\omega') f(\omega') (R_{\rho}\psi_{j})^{*}(\omega').$$
Spherical convolution
Rotated wavelet

Fast algorithms available

(e.g. McEwen et al. 2007, 2013, 2015).





Orthographic plot of spherical wavelets.

Scattering transform on the sphere

Spherical scattering propagator for scale *j*:

 $U[j]f = |f \star \psi_j|.$

Modulus function is adopted for the activation function (since non-expansive and preserves stability of wavelet representation).



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Spherical cascade of propagators:

$$U[p]f = |||f \star \psi_{j_1}| \star \psi_{j_2}| \ldots \star \psi_{j_d}|,$$

for the path $p = (j_1, j_2, \dots, j_d)$ with depth d.



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Scattering coefficients:

$$S[p]f = |||f \star \psi_{j_1}| \star \psi_{j_2}| \ldots \star \psi_{j_d}| \star \phi.$$



Scattering networks on the sphere

Spherical scattering network is collection of scattering transforms for a number of paths: $S_{\mathbb{P}}f = \{S[p]f : p \in \mathbb{P}\}, \text{ where the general path set } \mathbb{P} \text{ denotes the infinite set of all possible paths } \mathbb{P} = \{p = (j_1, j_2, \dots, j_d) : j_0 \le j_i \le J, 1 \le i \le d, d \in \mathbb{N}_0\}.$



Capture all information content at infinite depth and typically > 99% for depth d = 3.



Properties

Latent representation is very well-behaved and satisfies a number of important properties:

- 1. Rotational equivariance
- 2. Isometric invariance
- 3. Stability to diffeomorphisms



Rotationally equivariance

Rotational Equivariance

$$((\mathcal{R}_{\rho}f)\star\psi)(\rho')=(\mathcal{R}_{\rho}(f\star\psi))(\rho').$$





Rotationally equivariance

Rotational Equivariance

$$((\mathcal{R}_{\rho}f)\star\psi)(\rho')=(\mathcal{R}_{\rho}(f\star\psi))(\rho').$$





Isometric Invariance Let $\zeta \in \text{Isom}(\mathbb{S}^2)$, then there exists a constant C such that for all $f \in L^2(\mathbb{S}^2)$, $\|S_{\mathbb{P}_D}f - S_{\mathbb{P}_D}V_{\zeta}f\|_2 \leq CL^{5/2}(D+1)^{1/2} \frac{\lambda^{l_0}}{\|\zeta\|_{\infty}} \|f\|_2.$ Difference in representation. Scattering network representation is invariant to isometries up to a scale.



Isometric invariance





Stability to diffeomorphisms

Stability to Diffeomorphisms

Let $\zeta \in \text{Diff}(\mathbb{S}^2)$. If $\zeta = \zeta_1 \circ \zeta_2$ for some isometry $\zeta_1 \in \text{Isom}(\mathbb{S}^2)$ and diffeomorphism $\zeta_2 \in \text{Diff}(\mathbb{S}^2)$, then there exists a constant *C* such that for all $f \in L^2(\mathbb{S}^2)$,





Stability to diffeomorphisms



Stability to diffeomorphisms



Toy problem: Gaussianity of the cosmic microwave background (CMB)

Wavelet scattering as a representation space for classification.





 \rightsquigarrow 53% classification accuracy without scattering versus 95% with scattering.

Scattering for simulation-based inference (SBI)

Wavelet scattering as a representation space for SBI (Lin, Joachimi & McEwen 2024).





Spherical scattering covariance for generative modelling

Generative models of astrophysical fields with scattering transforms on the sphere (Mousset *et al.* McEwen 2024; s2scat code)

Scattering covariance statistics:

1.
$$S_1[\lambda] f = \mathbb{E} [|f \star \psi_{\lambda}|].$$

2. $S_2[\lambda] f = \mathbb{E} [|f \star \psi_{\lambda}|^2].$
3. $S_3[\lambda_1, \lambda_2] f = \operatorname{Cov} [f \star \psi_{\lambda_2}, |f \star \psi_{\lambda_1}| \star \psi_{\lambda_2}].$
4. $S_4[\lambda_1, \lambda_2, \lambda_3] f = \operatorname{Cov} [|f \star \psi_{\lambda_1}| \star \psi_{\lambda_3}, |f \star \psi_{\lambda_2}| \star \psi_{\lambda_3}].$


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4. $S_4[\lambda_1, \lambda_2, \lambda_3] f = \operatorname{Cov} [|f \star \psi_{\lambda_1}| \star \psi_{\lambda_3}, |f \star \psi_{\lambda_2}| \star \psi_{\lambda_3}]$

Generative modelling by matching set of scattering covariance statistics S(f) with a (single) target simulation:

$$\min_{f} \|\mathcal{S}(f) - \mathcal{S}(f_{\text{target}})\|^2.$$



Differentiable and GPU-accelerated spherical transform codes (in JAX)

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Differentiable and accelerated spherical transforms

SEFFT is a Python package for computing Fourier transforms on the sphere and rotation group (<u>Price & McEwen</u> 2023) using JAX or PyTorch. It leverages autodiff to provide differentiable transforms, which are also deployable on hardware accelerators (e.g. GPUs and TPUs).

s2fft: Spherical harmonic transforms https://github.com/astro-informatics/s2fft

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Differentiable and accelerated wavelet transform on the sphere

S2MAY is a python package for computing wavelet transforms on the sphere and rotation group, both in JAX and PyTorch. It leverages autodiff to provide differentiable transforms, which are also deployable on modern hardware accelerators (s.g. GPUs and TPUs), and can be mapped across multiple accelerators.

s2wav: Spherical wavelet transforms https://github.com/astro-informatics/s2wav

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Differentiable scattering covariances on the sphere

SSEAT is a Python package for computing scattering covariances on the sphere (<u>Mousset et al. 2024</u>) using JAX. It exploits autodiff to provide differentiable transforms, which are also deployable on hardware accelerators (e.g. GPUs and TPUs), leveraging the differentiable and accelerated spherical harmonic and wavelet transforms implemented in SZFF1 and SZWAV, respectively.

s2scat: Spherical scattering transforms
https://github.com/astro-informatics/s2scat

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Scalable and Equivariant Spherical CNNs by Discrete-Continuous (DISCO) Convolutions

Many problems across computer vision and the natural sciences require the analysis of spherical data, for which representations may be learned efficiently by encoding equivariance to rotational symmetries. (DISCO, provides foundational convolutional layers which encode eail equivariance, with the aim to support the development of

s2ai: Spherical AI

Coming very soon! Contact us for early access.



Generative modelling of large scale structure (LSS)

Which field is emulated and which simulated?



Logarithm (for visualization) of weak lensing field.



Generative modelling of large scale structure (LSS)





Generative modelling of cosmic strings in the CMB

Need to **simulate full physics**, evolving a network of strings through cosmic time, and then ray-trace CMB photons through the string network (Ringeval et al. 2012).

A single simulation requires 800,000 CPU hours on a supercomputer.

There are **only three full-sky string maps in existence**.





Generative modelling of cosmic strings in the CMB

Computation time: 800,000 CPU hours on supercomputer $\rightarrow O(1)$ hours on A100 GPU.

Still work in progress (statistical validation in progress).



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Summary







With great power comes great responsibility!