Wavelet reconstruction of E- and B-modes for weak lensing mass mapping and CMB polarisation

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arXiv:1605.01414

In collaboration with Boris Leistedt, Martin Büttner & Hiranya Peiris

Mapping the Cosmic Web, Royal Astronomical Society (RAS), London, June 2016

Outline

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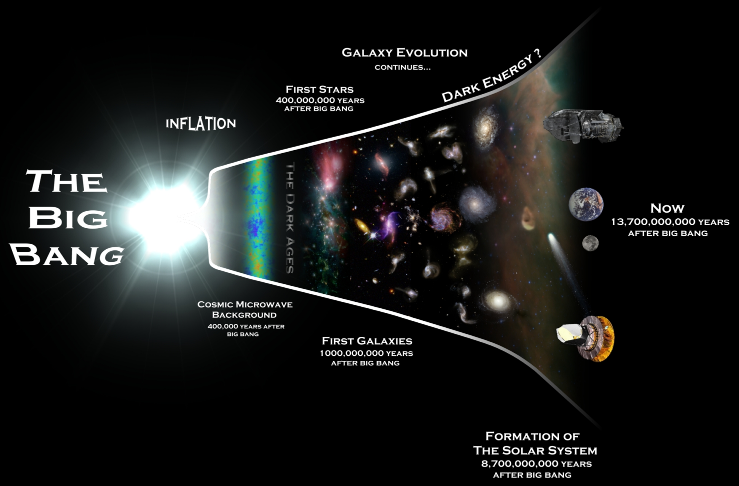
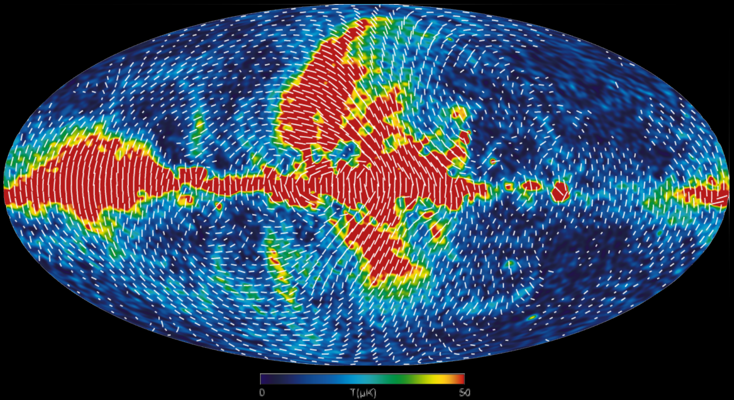
Unanswered fundamental questions



[Credit: Rhys Taylor]

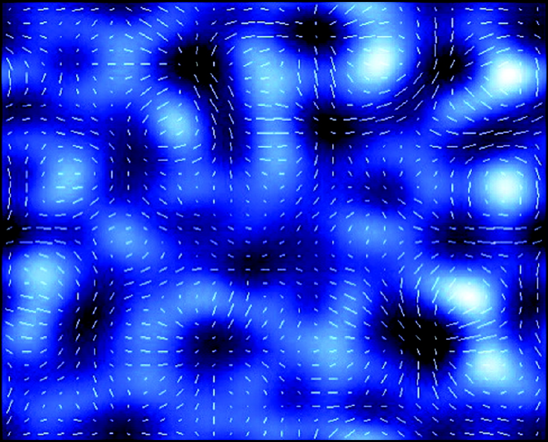
CMB polarisation



WMAP K-band ${}_2P = Q + \mathrm{i}U$ map

[Credit: WMAP]

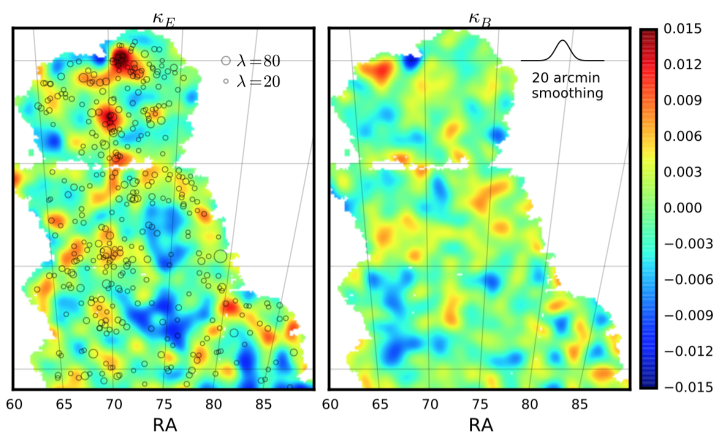
Cosmic shear



Cosmic shear ${}_2\gamma = \gamma_1 + \mathrm{i}\gamma_2$ map

[Credit: Ellis (2010)]

Convergence from cosmic shear



Convergence map of DES science verification data

[Credit: Chang *et al.* (2015)]

Cosmological spin signals

- Observe spin ± 2 cosmological signals on the celestial sphere, with $\boldsymbol{n} = (\theta, \varphi) \in \mathbb{S}^2$.
  - CMB polarisation: $${}_{\pm 2}P(\boldsymbol{n}) = Q \pm iU$$
  - Cosmic shear: $${}_{\pm 2}\gamma(\boldsymbol{n}) = \gamma_1 \pm i\gamma_2$$
- Dependent on choice of local coordinate frame.
- Spin ± 2 signals transform under local rotations of χ by, *e.g.*,

$${}_{\pm 2}P' = \mathrm{e}^{\mp \mathrm{i}2\chi} \; {}_{\pm 2}P \, .$$

- To confront cosmological models with observations, transform observable spin signals to scalar (and pseudo-scalar) signals, which are invariant to choice of local coordinate frame.

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E- and B-modes

Full-sky

- Decompose $_{\pm 2}P$ into parity even and parity odd components:

$$\epsilon(\boldsymbol{n}) = -\frac{1}{2}\left[\bar{\eth}^2 \; {}_2P(\boldsymbol{n}) + \eth^2 \; {}_{-2}P(\boldsymbol{n})\right] \quad \text{E-mode}$$

$$\beta(\boldsymbol{n}) = \frac{\mathrm{i}}{2}\left[\bar{\eth}^2 \; {}_2P(\boldsymbol{n}) - \eth^2 \; {}_{-2}P(\boldsymbol{n})\right] \quad \text{B-mode}$$

where $\bar{\eth}$ and $\eth$ are spin lowering and raising (differential) operators, respectively.

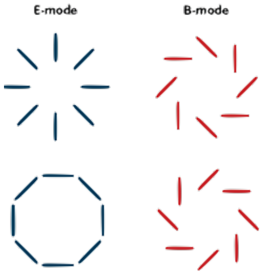

Figure: E-mode (even parity) and B-mode (odd parity) signals [Credit: http://www.skyandtelescope.com/].

- Different physical processes exhibit different symmetries and thus behave differently under parity transformation.
- Can exploit this property to separate signals arising from different underlying physical mechanisms.
- Mapping E- and B-modes on the sky of great importance for forthcoming experiments.

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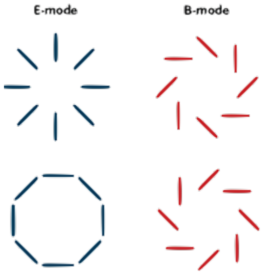

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E- and B-modes

Partial-sky

- On a manifold without boundary (*i.e.* full sky), a spin ± 2 signal can be decomposed uniquely into E- and B-modes.
- On a manifold with boundary (*i.e.* partial sky), decomposition not unique.
- Recovering E and B-modes from partial sky observations is challenging since mask leaks contamination.
- Pure and ambiguous modes (Lewis *et al.* 2002, Bunn *et al.* 2003, Smith 2006, Smith & Zaldarriaga 2007, Grain *et al.* 2007, Ferté *et al.* 2013).
  - E-modes: vanishing curl
  - B-modes: vanishing divergence
  - Pure E-modes: orthogonal to all B-modes
  - Pure B-modes: orthogonal to all E-modes
- Number of existing techniques (Lewis *et al.* 2002, Bunn *et al.* 2003, Smith 2006, Smith & Zaldarriaga 2007, Grain *et al.* 2007, Bowyer *et al.* 2011, Kim 2013, Ferté *et al.* 2013).
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1. E- and B-modes
2. Spin wavelets
3. E/B separation

Spin scale-discretised wavelets on the sphere
Wavelet construction

- Directional spin wavelets on the sphere (McEwen *et al.* 2015; arXiv:1509.06749)
  - Generalise scale-discretised wavelets (Wiaux, McEwen, Vandergheynst, Blanc 2008) to signals of arbitrary arbitrary spin.

- Spin scale-discretised wavelet ${}_s\Psi^j$ constructed in harmonic space:

$$ {}_s\Psi^j_{\ell m} = \kappa^j(\ell)\, \zeta_{\ell m} . $$

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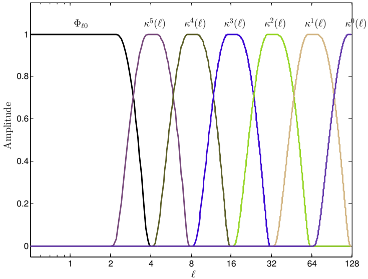

Figure: Harmonic tiling on the sphere.

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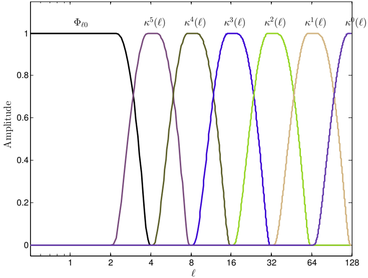

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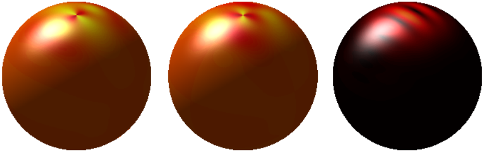

(a) Real$({}_s\Psi^j)$ (b) Imag$({}_s\Psi^j)$ (c) Abs$({}_s\Psi^j)$

Figure: Spin scale-discretised wavelets on the sphere.

Spin scale-discretised wavelets on the sphere

Forward transform (i.e. analysis)

- The spin scale-discretised wavelet transform is given by projection onto each wavelet:

$$\boxed{W^{{}_s\Psi^j}_{{}_sP}(\rho) = \langle {}_sP,\, \mathcal{R}_\rho\, {}_s\Psi^j \rangle} = \int_{\mathbb{S}^2} \mathrm{d}\Omega(\boldsymbol{n})\, {}_sP(\boldsymbol{n}) (\mathcal{R}_\rho\, {}_s\Psi^j)^*(\boldsymbol{n})\ ,$$

projection

where $\mathrm{d}\Omega(\boldsymbol{n}) = \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\varphi$, and rotations parameterised by $\rho = (\alpha, \beta, \gamma) \in \mathrm{SO}(3)$.

- Wavelet coefficients for scale j live on rotation group $\mathrm{SO}(3)$

  $\Rightarrow$ directional structure is naturally incorporated.

- Other wavelet transforms on the sphere:
  - Stereographic projection (Antoine & Vandergheynst 1999, Wiaux *et al.* 2005)
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Spin scale-discretised wavelets on the sphere

Inverse transform (i.e. synthesis)

- Original signal may be recovered exactly from its wavelet coefficients:

$$ {}_sP(\boldsymbol{n}) = \boxed{\sum_{j=0}^{J}} \boxed{\int_{\mathrm{SO}(3)} \mathrm{d}\varrho(\rho) W^{{}_s\Psi^j}_{{}_sP}(\rho) (\mathcal{R}_\rho \, {}_s\Psi^j)(\boldsymbol{n})} , $$

finite sum — wavelet contribution

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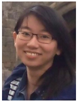
- Other types of scale-discretised wavelets:
  - Curvelets (Chan *et al.* 2015; arXiv:1511.05578)



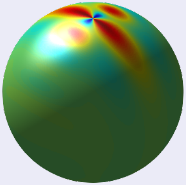
  - Ridgelets (McEwen 2015; arXiv:1510.01595).
  - Spin flaglets on the 3D ball (Leistedt *et al.* 2015; arXiv:1509.06750)

Spin scale-discretised wavelets on the sphere

Codes (www.jasonmcewen.org/codes.html)

S2LET code

http://www.s2let.org

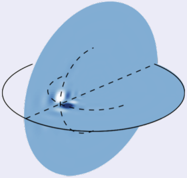


S2LET: Fast & exact wavelets on the sphere
Leistedt, McEwen, Vandergheynst, Wiaux (2012)
McEwen, Leistedt, Büttner, Peiris, Wiaux (2015)

- C, Matlab, Python, IDL
- Supports directional, steerable, spin wavelets
- Fast algos

FLAGLET code

http://www.flaglets.org



FLAGLET: Fast & exact wavelets on the ball
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E/B separation

Connections between spin and scalar wavelet coefficients

- Spin wavelet transform of ${}_{\pm2}P = Q \pm \mathrm{i}U$ (observable):

$$W^{{}_2\Psi^j}_{{}_{\pm2}P}(\rho) = \langle {}_{\pm2}P,\, \mathcal{R}_\rho\, {}_{\pm2}\Psi^j \rangle = \int_{\mathbb{S}^2} \mathrm{d}\Omega(\boldsymbol{n})\, {}_{\pm2}P(\boldsymbol{n}) (\mathcal{R}_\rho\, {}_{\pm2}\Psi^j)^*(\boldsymbol{n}) \,.$$

spin wavelet transform

- Scalar wavelet transforms of E and B (non-observable):

$$W^{{}_0\Psi^j}_{\epsilon}(\rho) = \langle \epsilon,\, \mathcal{R}_\rho\, {}_0\Psi^j \rangle \,,$$

scalar wavelet transform

$$W^{{}_0\Psi^j}_{\beta}(\rho) = \langle \beta,\, \mathcal{R}_\rho\, {}_0\Psi^j \rangle \,,$$

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where ${}_0\Psi^j \equiv \bar{\eth}^2\, {}_2\Psi^j$.

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E/B separation
Exploiting wavelets

General approach to recover E/B signals using scale-discretised wavelets

1. Compute spin wavelet transform of ${}_{\pm2}P = Q + \mathrm{i}U$:

$$ {}_{\pm2}P(\boldsymbol{n}) \xrightarrow[\text{S2LET}]{\text{Spin wavelet transform}} W^{{}_2\Psi^j}_{{}_{\pm2}P}(\rho) $$

2. Account for mask in wavelet domain (simultaneous harmonic and spatial localisation):

$$ W^{{}_2\Psi^j}_{{}_{\pm2}P}(\rho) \xrightarrow{\text{Mitigate mask}} \bar{W}^{{}_2\Psi^j}_{{}_{\pm2}P}(\rho) $$

3. Construct E/B maps:

(a) $$ W^{{}_0\Psi^j}_{\epsilon}(\rho) = -\mathrm{Re}\left[\bar{W}^{{}_2\Psi^j}_{{}_{\pm2}P}(\rho)\right] \xrightarrow[\text{S2LET}]{\text{Inverse scalar wavelet transform}} \epsilon(\boldsymbol{n}) $$

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2. Account for mask in wavelet domain (simultaneous harmonic and spatial localisation):

$$ W^{{}_2\Psi^j}_{{}_{\pm 2}P}(\rho) \xrightarrow{\text{Mitigate mask}} \bar{W}^{{}_2\Psi^j}_{{}_{\pm 2}P}(\rho) $$

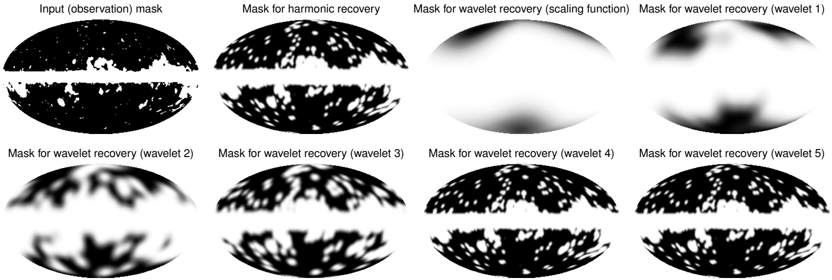
3. Construct E/B maps:

(a) $W^{{}_0\Psi^j}_{\epsilon}(\rho) = -\mathrm{Re}\left[\bar{W}^{{}_2\Psi^j}_{{}_{\pm 2}P}(\rho)\right] \xrightarrow[\text{S2LET}]{\text{Inverse scalar wavelet transform}} \epsilon(\boldsymbol{n})$

(b) $W^{{}_0\Psi^j}_{\beta}(\rho) = \mp\mathrm{Im}\left[\bar{W}^{{}_2\Psi^j}_{{}_{\pm 2}P}(\rho)\right] \xrightarrow[\text{S2LET}]{\text{Inverse scalar wavelet transform}} \beta(\boldsymbol{n})$

E/B separation

Scale-dependent masking



E/B separation

Pure mode wavelet estimator

- Consider masked Stokes parameters:

$$ {}_0M = M, \quad {}_{\pm1}M = ð_\pm M, \quad {}_{\pm2}M = ð^2_\pm M, $$

spin adjusted masks

$$ {}_{\pm2}\widetilde{P} = {}_0M\,{}_{\pm2}P, \quad {}_{\pm1}\widetilde{P} = {}_{\mp1}M\,{}_{\pm2}P, \quad {}_{\pm0}\widetilde{P} = {}_{\mp2}M\,{}_{\pm2}P. $$

masked Stokes parameters

where $ð_\pm = \{\, ð \text{ if } + \,,\ \bar{ð} \text{ if } - \,\}$.

- Pure wavelet estimators (see Leistedt, McEwen, Büttner, Peiris 2016; arXiv:1605.01414):

$$ \widehat{W}^{0\Psi^j}_{\epsilon}(\rho) = -\operatorname{Re}\left[W^{\pm2\Upsilon^j}_{\pm2\widetilde{P}}(\rho) + 2W^{\pm1\Upsilon^j}_{\pm1\widetilde{P}}(\rho) + W^{0\Upsilon^j}_{0\widetilde{P}}(\rho) \right], $$

pure E

$$ \widehat{W}^{0\Psi^j}_{\beta}(\rho) = \mp\operatorname{Im}\left[W^{\pm2\Upsilon^j}_{\pm2\widetilde{P}}(\rho) + 2W^{\pm1\Upsilon^j}_{\pm1\widetilde{P}}(\rho) + W^{0\Upsilon^j}_{0\widetilde{P}}(\rho) \right], $$

pure B

where ${}_{\pm s}\Upsilon^j = ð^s_\pm({}_0\Psi^j)$ are spin adjusted wavelets and assuming the Dirichlet and Neumann boundary conditions, *i.e.* that the mask and its derivative vanish at the boundaries.

E/B separation

Pure mode wavelet estimator

- Consider masked Stokes parameters:

$$ {}_0M = M, \quad {}_{\pm1}M = \eth_{\pm} M, \quad {}_{\pm2}M = \eth_{\pm}^2 M, $$

spin adjusted masks

$$ {}_{\pm2}\widetilde{P} = {}_0M \, {}_{\pm2}P, \quad {}_{\pm1}\widetilde{P} = {}_{\mp1}M \, {}_{\pm2}P, \quad {}_{\pm0}\widetilde{P} = {}_{\mp2}M \, {}_{\pm2}P. $$

masked Stokes parameters

where $\eth_{\pm} = \{ \eth \text{ if } + , \bar{\eth} \text{ if } - \}$.

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pure E

$$ \widehat{W}_{\beta}^{0\Psi^j}(\rho) = \mp \,\mathrm{Im}\left[W^{\pm2\Upsilon^j}_{\pm2\widetilde{P}}(\rho) + 2W^{\pm1\Upsilon^j}_{\pm1\widetilde{P}}(\rho) + W^{0\Upsilon^j}_{0\widetilde{P}}(\rho) \right], $$

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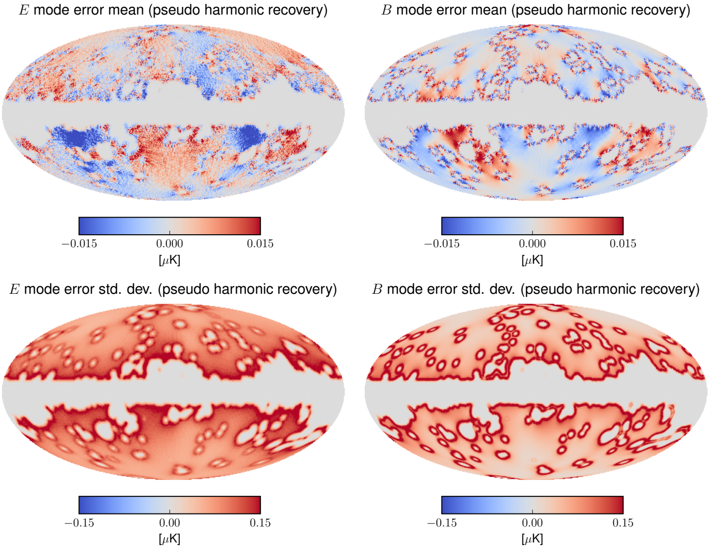
- Correction terms require spin ±1 wavelet transforms.

E/B separation

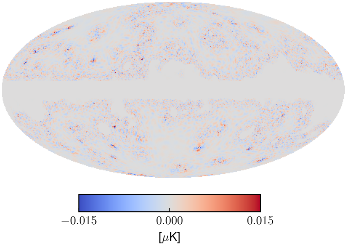
Results: pseudo harmonic approach



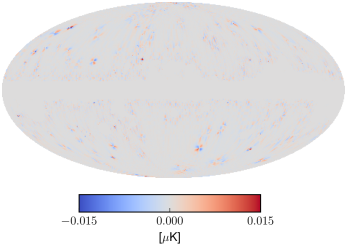
E/B separation

Results: pure wavelet approach

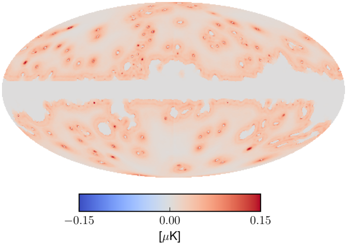
E mode error mean (pure wavelet recovery)



B mode error mean (pure wavelet recovery)



E mode error std. dev. (pure wavelet recovery)



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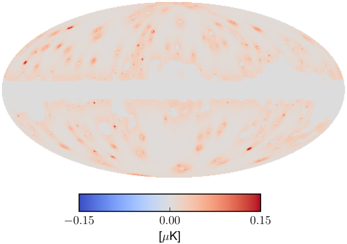


Summary

- Pure E/B separation with spin wavelets (without optimisation) reduces leakage by over an order of magnitude (Leistedt *et al.* 2016; arXiv:1605.01414).
- Improvement in sensitivity to tensor-to-scalar ratio r of 10^2–10^4.
- Problem for weak lensing is completely analogous. Applying to mass mapping. . .
- Future extensions:
  - Optimise wavelet parameters
  - Optimal masks
  - Exploit directionality

Spin scale-discretised wavelets are a powerful tool
for weak lensing and CMB polarisation.

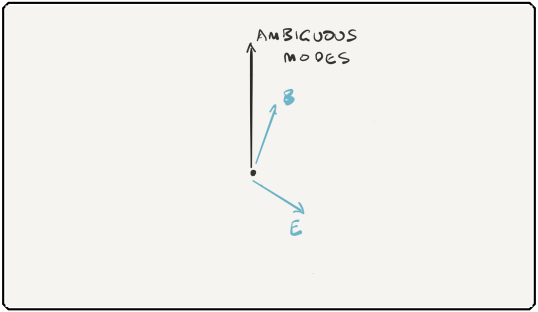
www.s2let.org
www.jasonmcewen.org

Extra Slides

E- and B-modes

Pure and ambiguous modes

- Pure and ambiguous modes
  (Lewis *et al.* 2002, Bunn *et al.* 2003, Smith 2006, Smith & Zaldarriaga 2007, Grain *et al.* 2007, Ferté *et al.* 2013)
  - E-modes: vanishing curl
  - B-modes: vanishing divergence
  - Pure E-modes: orthogonal to all B-modes
  - Pure B-modes: orthogonal to all E-modes



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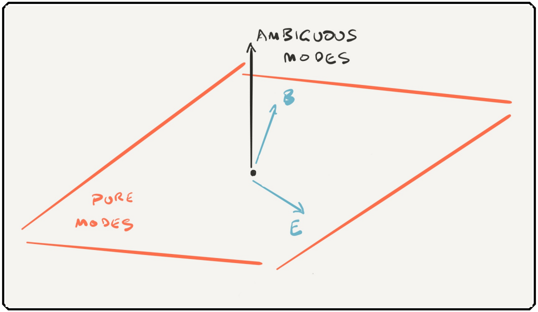

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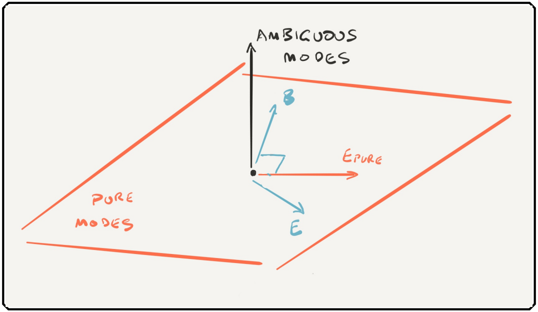

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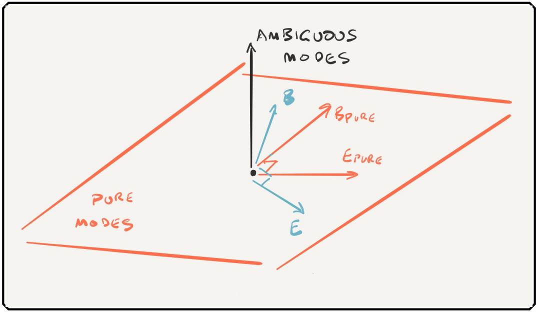

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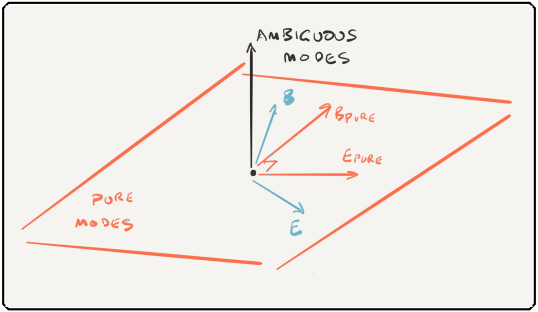

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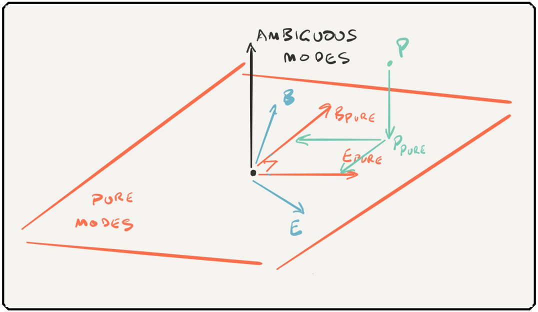
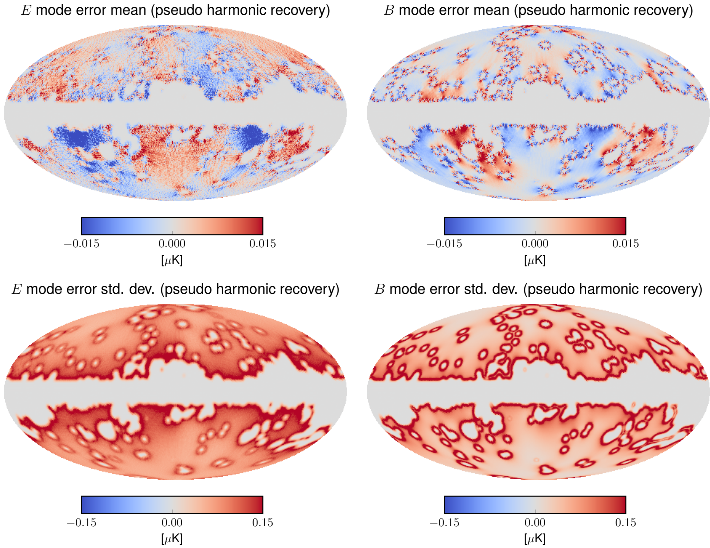


E/B separation

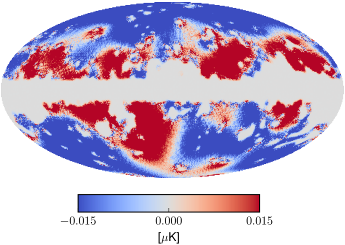
Results: pseudo harmonic approach



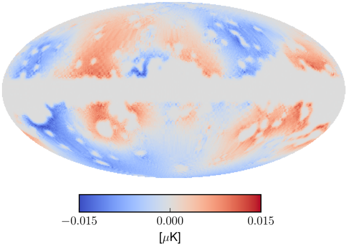
E/B separation

Results: pure harmonic approach

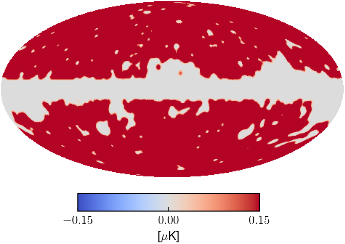
E mode error mean (pure harmonic recovery)



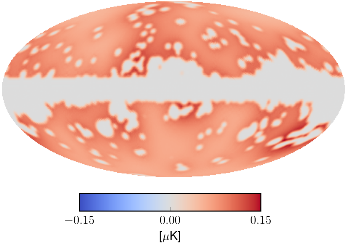
B mode error mean (pure harmonic recovery)



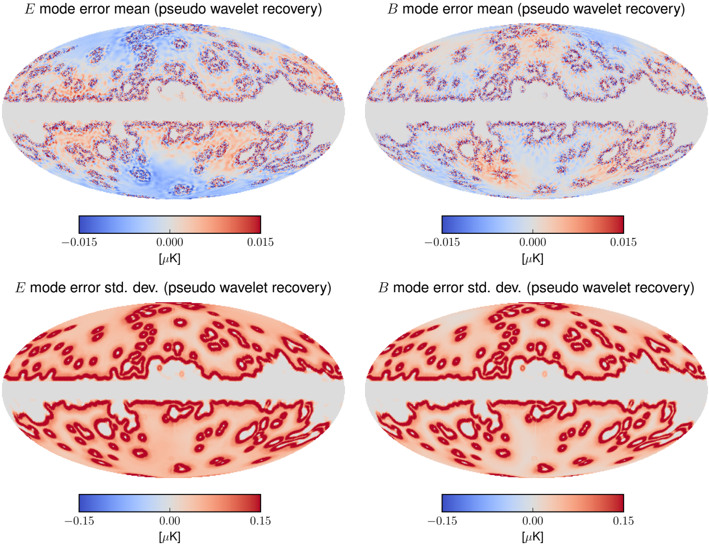
E mode error std. dev. (pure harmonic recovery)



B mode error std. dev. (pure harmonic recovery)



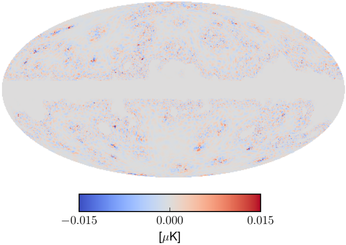
E/B separation
Results: pseudo wavelet approach



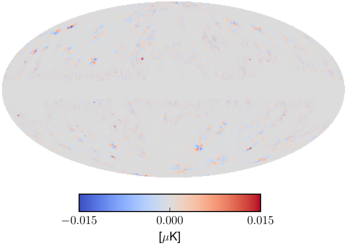
E/B separation

Results: pure wavelet approach

E mode error mean (pure wavelet recovery)



B mode error mean (pure wavelet recovery)



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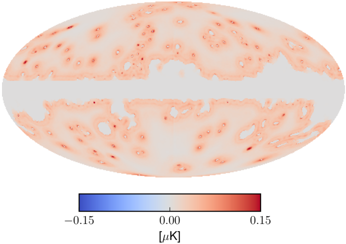

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