

Bianchi VII_h cosmologies and *Planck*

XXVI. Background geometry and topology of the Universe

Planck Collaboration

Presented by Jason McEwen

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University College London (UCL)

The Universe as seen by Planck :: 47th ESLAB Symposium :: April 2013



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Outline

- 1 Topology
- 2 Bianchi VII_h cosmologies
- 3 Bayesian analysis
- 4 *Planck* results

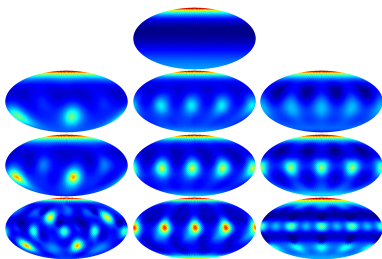


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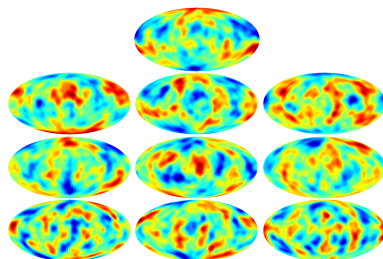
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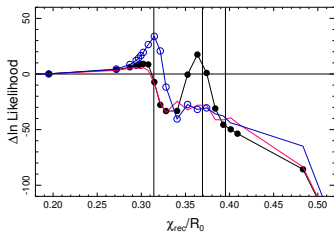


(a) Correlations

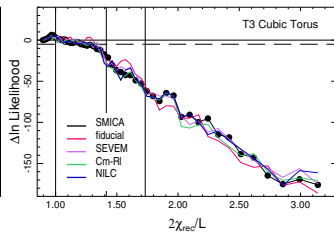


(b) Simulations

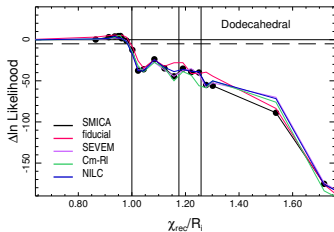
Topology



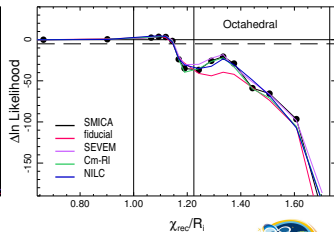
(a) Simulation



(b) T3 Cubic Torus



(c) Dodecahedral

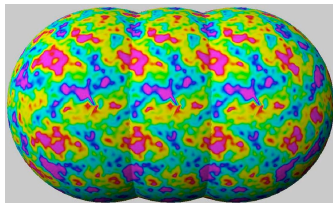


(d) Octahedral

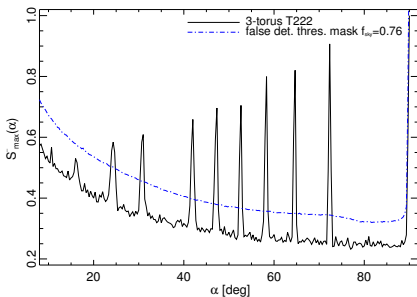


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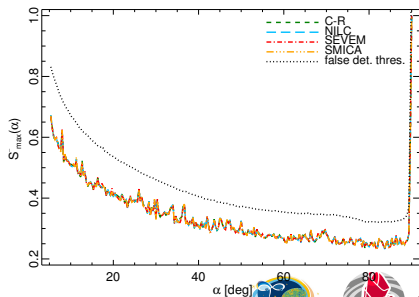
Topology



(a) Back-to-back circles



(b) Simulation



(c) Planck data



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Outline

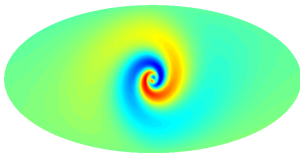
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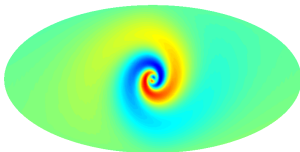
Bianchi VII_h cosmologies

- Relax assumptions about the global structure of spacetime by **allowing anisotropy** about each point in the Universe.
- Yields more general solutions to Einstein's field equations → **Bianchi cosmologies**.
- For small anisotropy, as already demanded by current observations, linear perturbation about the standard FRW model may be applied.
- Induces a characteristic subdominant, **deterministic signature in the CMB**, which is **embedded in the usual stochastic anisotropies**.
- First examined by Collins & Hawking (1973) and Barrow *et al.* (1985), however dark energy not included.
- Focus on **Bianchi VII_h**, using solutions derived by Anthony Lasenby that do incorporate dark energy (also derived independently by Jaffe *et al.* 2006).



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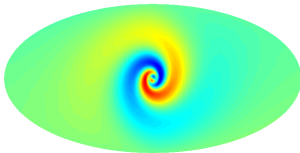
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Bianchi VII_h cosmologies

- **Bianchi VII_h** models describe a universe with overall **rotation**, with angular velocity ω , and a three-dimensional rate of **shear**, specified by the antisymmetric tensor σ_{ij} . Throughout we assume equality of shear modes $\sigma = \sigma_{12} = \sigma_{13}$ (cf. Jaffe *et al.* 2005).
- The **amplitude** of induced CMB temperature fluctuations may be characterised by the **dimensionless vorticity** $(\omega/H)_0$, which influences the amplitude of the induced temperature contribution only and not its morphology.
- The model has a **free parameter**, denoted x , describing the **comoving length-scale over which the principal axes of shear and rotation change orientation**.
- The **orientation** and **handedness** of the coordinate system is also free.
- Bianchi VII_h models may be described by the parameter vector:

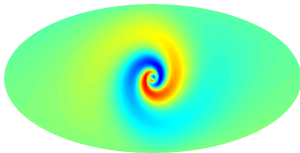
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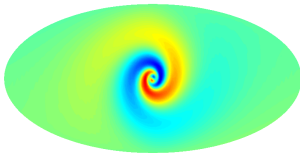
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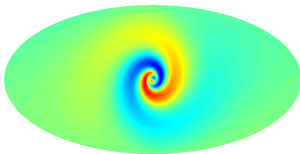
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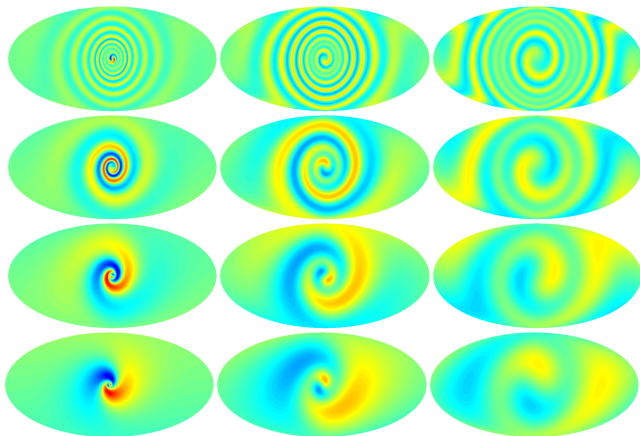


Figure: Simulated deterministic CMB temperature contributions in Bianchi VII_h cosmologies for varying x and Ω_{total} (left-to-right $\Omega_{\text{total}} \in \{0.10, 0.30, 0.95\}$; top-to-bottom $x \in \{0.1, 0.3, 0.7, 1.5\}$).

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Bayesian analysis of Bianchi VII_h cosmologies

- Perform the **Bayesian analysis** described by
JDM, **Thibaut Josset**, **Stephen Feeney**, Hiranya Peiris, Anthony Lasenby (2013)
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and applied to WMAP previously.

- **Posterior distribution** of the parameters Θ of model of interest M given data d , as

$$P(\Theta | d, M) \propto P(d | \Theta, M) P(\Theta | M) .$$

- Consider open and flat cosmologies with cosmological parameters:

$$\Theta_C = (A_s, n_s, \tau, \Omega_b h^2, \Omega_c h^2, \Omega_\Lambda, \Omega_k).$$

- Recall Bianchi parameters:

$$\Theta_B = (\Omega_m, \Omega_\Lambda, x, (\omega/H)_0, \alpha, \beta, \gamma).$$

- **Likelihood** is given by

$$P(d | \Theta_B, \Theta_C) \propto \frac{1}{\sqrt{|X(\Theta_C)|}} e^{\left[-\chi^2(\Theta_C, \Theta_B)/2 \right]} ,$$

where

$$\chi^2(\Theta_C, \Theta_B) = [d - b(\Theta_B)]^\dagger X^{-1}(\Theta_C) [d - b(\Theta_B)] .$$

- Consider **decoupled (phenomenological)** and **coupled (physical)** analyses.



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Bayesian analysis of Bianchi VII_h cosmologies

- Bianchi VII_h templates can be computed accurately and rotated efficiently in harmonic space
→ consider **harmonic space representation**, where $\mathbf{d} = \{d_{\ell m}\}$ and $\mathbf{b}(\Theta_B) = \{b_{\ell m}(\Theta_B)\}$.
- **Partial-sky analysis** that handles in harmonic space a mask applied in pixel space.
- Add **masking noise** in order to marginalise the pixel values of the data contained in the masked region, with variance for pixel i given by

$$\sigma_m^2(\omega_i) = \begin{cases} \Sigma_m^2, & \omega_i \in \mathbb{M} \\ 0, & \omega_i \in \mathbb{S}^2 \setminus \mathbb{M} \end{cases},$$

where Σ_m^2 is a constant masking noise variance.

- The **covariance** is then given by

$$\mathbf{X}(\Theta_C) = \mathbf{C}(\Theta_C) + \mathbf{M},$$

where

- $\mathbf{C}(\Theta_C)$ is the diagonal CMB covariance defined by the power spectrum $C_\ell(\Theta_C)$;
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$$\mathbf{M}_{\ell m \ell' m'}^{\ell' m'} = \langle m_{\ell m} m_{\ell' m'}^* \rangle \simeq \sum_{\omega_i} \sigma_m^2(\omega_i) Y_{\ell m}^*(\omega_i) Y_{\ell' m'}(\omega_i) \Omega_{\text{pix}}^2.$$



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Bayesian analysis of Bianchi VII_h cosmologies

- Compute the **Bayesian evidence** to determine preferred model:

$$E = P(\mathbf{d} | M) = \int d\Theta P(\mathbf{d} | \Theta, M) P(\Theta | M) .$$

- Use **MultiNest** to compute the posteriors and evidences via nested sampling (Feroz & Hobson 2008, Feroz *et al.* 2009).
- Consider two models:
 - **Flat-decoupled-Bianchi** model: Θ_C and Θ_B fitted simultaneously but **decoupled**
→ phenomenological
 - **Open-coupled-Bianchi** model: Θ_C and Θ_B fitted simultaneously and **coupled**
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Validation with simulations

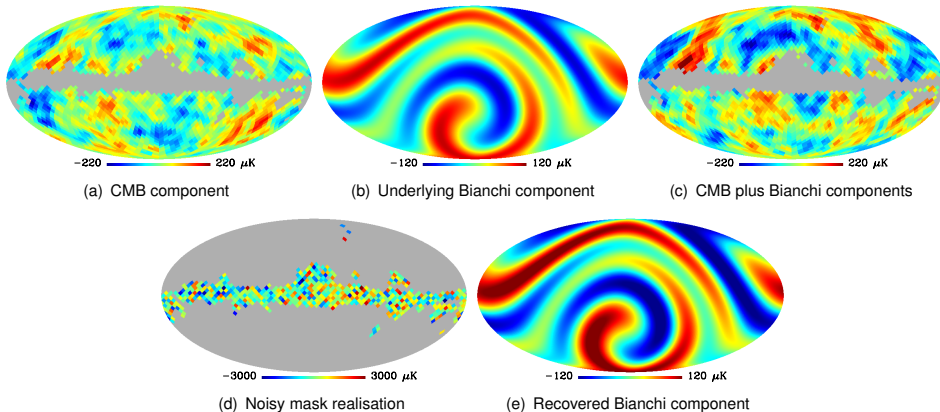


Figure: Partial-sky simulation with embedded Bianchi VII_h component at $\ell_{\text{max}} = 32$.

Validation with simulations

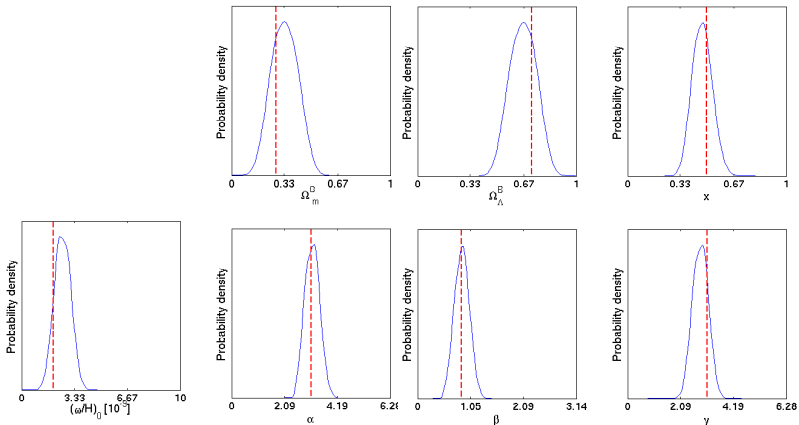


Figure: Marginalised posterior distributions recovered from partial-sky simulation at $\ell_{\max} = 32$.

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Planck results: flat-decoupled-Bianchi model

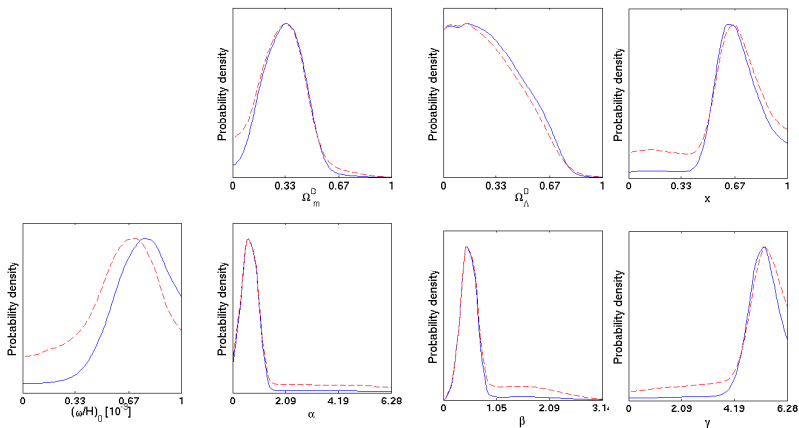


Figure: Posterior distributions of Bianchi parameters recovered for the **phenomenological** flat-decoupled-Bianchi model from *Planck* **SMICA** (solid curves) and **SEVEM** (dashed curves) data.

Planck results: flat-decoupled-Bianchi model

Table: Bayes factor relative to equivalent Λ CDM model (positive favours Bianchi model).

Model	$\Delta \ln E$	
	SMICA	SEVEM
Flat-decoupled-Bianchi (left-handed)	2.8 ± 0.1	1.5 ± 0.1
Flat-decoupled-Bianchi (right-handed)	0.5 ± 0.1	0.5 ± 0.1

- On the Jeffreys (1961) scale, **evidence** for the inclusion of a Bianchi VII_h component would be termed **strong** (significant) for SMICA (SEVEM) component-separated data.
- A log-Bayes factor of 2.8 corresponds to an odds ratio of approximately 1 in 16.
- **Planck data favour the inclusion of a phenomenological Bianchi VII_h component.**
- Best-fit Bianchi VII_h **template is similar to that first found in WMAP** data by Jaffe *et al.* 2005.

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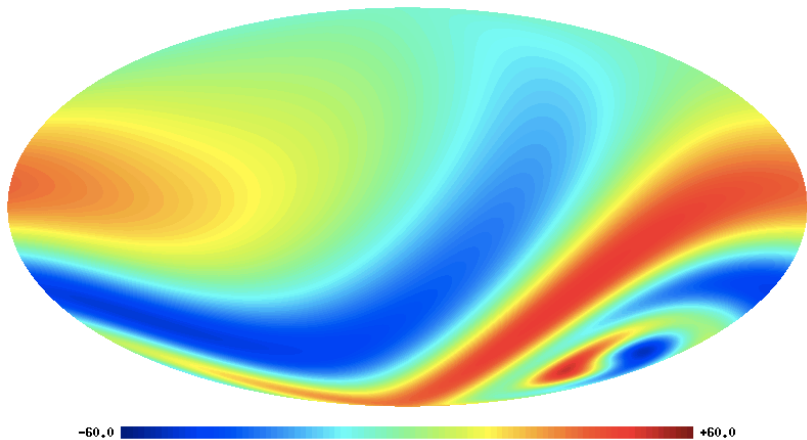


Figure: Best-fit template of flat-decoupled-Bianchi VII_h model found in *Planck* SMICA component-separated data.

Planck results: flat-decoupled-Bianchi model

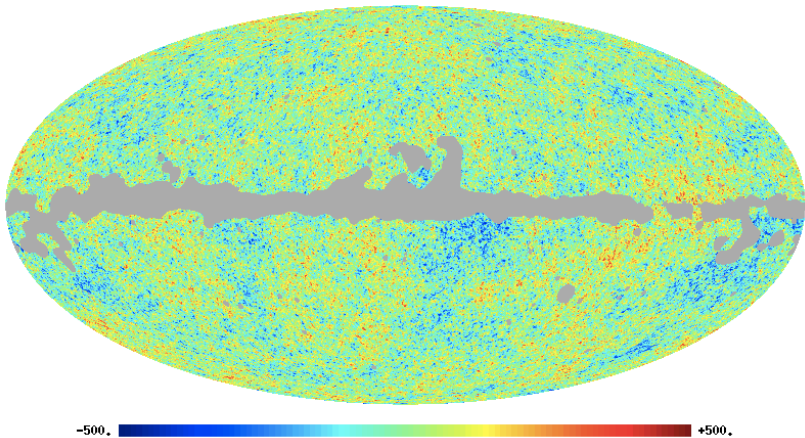


Figure: *Planck* SMICA component-separated data.

Planck results: flat-decoupled-Bianchi model

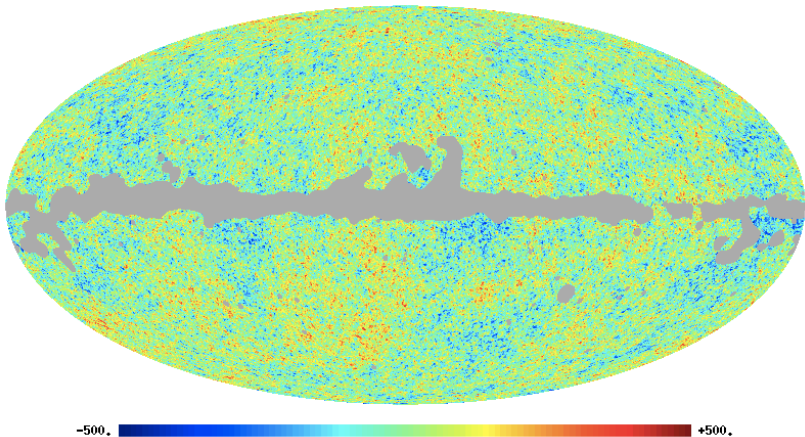


Figure: *Planck* SMICA component-separated data minus best-fit template of flat-decoupled-Bianchi VII_h model.

Planck results: flat-decoupled-Bianchi model

- BUT** the flat-Bianchi-decoupled model is phenomenological and **not physical!**

Table: Parameters recovered for flat-decoupled-Bianchi model.

Bianchi Parameter	SMICA		SEVEM	
	MAP	Mean	MAP	Mean
$\Omega_{\text{m}}^{\text{B}}$	0.38	0.32 ± 0.12	0.35	0.31 ± 0.15
$\Omega_{\Lambda}^{\text{B}}$	0.20	0.31 ± 0.20	0.22	0.30 ± 0.20
x	0.63	0.67 ± 0.16	0.66	0.62 ± 0.23
$(\omega/H)_0$	8.8×10^{-10}	$(7.1 \pm 1.9) \times 10^{-10}$	9.4×10^{-10}	$(5.9 \pm 2.4) \times 10^{-10}$
α	38.8°	$51.3^{\circ} \pm 47.9^{\circ}$	40.5°	$77.4^{\circ} \pm 80.3^{\circ}$
β	28.2°	$33.7^{\circ} \pm 19.7^{\circ}$	28.4°	$45.6^{\circ} \pm 32.7^{\circ}$
γ	309.2°	$292.2^{\circ} \pm 51.9^{\circ}$	317.0°	$271.5^{\circ} \pm 80.7^{\circ}$

Planck results: open-coupled-Bianchi model

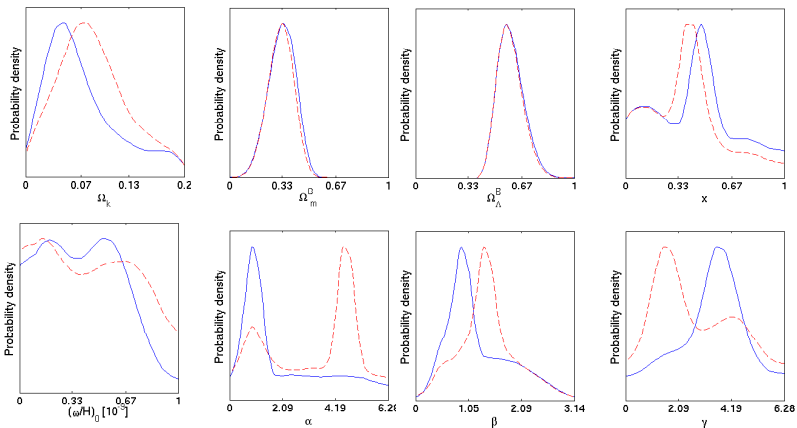


Figure: Posterior distributions of Bianchi parameters recovered for the **physical** open-coupled-Bianchi model from *Planck* **SMICA** (solid curves) and **SEVEM** (dashed curves) data.

Planck results: open-coupled-Bianchi model

Table: Bayes factor relative to equivalent Λ CDM model (positive favours Bianchi model).

Model	$\Delta \ln E$	
	SMICA	SEVEM
Open-coupled-Bianchi (left-handed)	0.0 ± 0.1	0.0 ± 0.1
Open-coupled-Bianchi (right-handed)	-0.4 ± 0.1	-0.4 ± 0.1

- In the physical setting where the standard cosmological and Bianchi parameters are coupled, *Planck data do not favour the inclusion of a Bianchi VII_i component.*
- We find **no evidence for Bianchi VII_i cosmologies** and constrain the vorticity of such models to $(\omega/H)_0 < 8.1 \times 10^{-10}$ (95% confidence level).

Summary

- Perform a **Bayesian analysis** of **partial-sky Planck data** for evidence of Bianchi VII_h cosmologies.
- Planck data support the inclusion of a **phenomenological Bianchi template**...
- **BUT** this model is **non-physical** and the recovered **cosmological parameters are inconsistent with standard constraints!**
- In the **physical model** where the cosmological and Bianchi parameters are coupled, **Planck data do not favour the inclusion of a Bianchi VII_h component**.
- We **constrain vorticity** of Bianchi VII_h cosmologies to $(\omega/H)_0 < 8.1 \times 10^{-10}$ (95% confidence level).

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Summary

- Perform a **Bayesian analysis** of **partial-sky Planck data** for evidence of Bianchi VII_h cosmologies.
- Planck data support the inclusion of a **phenomenological Bianchi template**...
- **BUT** this model is **non-physical** and the recovered **cosmological parameters are inconsistent with standard constraints!**
- In the **physical model** where the cosmological and Bianchi parameters are coupled, **Planck data do not favour the inclusion of a Bianchi VII_h component.**
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The scientific results that we present today are a product of the Planck Collaboration, including individuals from more than 100 scientific institutes in Europe, the USA and Canada



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