Next-generation radio interferometric imaging with compressive sensing

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Outline

1. Radio Interferometry (RI)
2. Compressive Sensing (CS)
3. Radio Interferometric Imaging with Compressive Sensing (RI+CS)
4. Spread Spectrum
5. Continuous Visibilities
6. Outlook

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4. Spread Spectrum
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Next-generation of radio interferometry rapidly approaching

- Square Kilometre Array (SKA) first observations planned for 2019.
- Many other pathfinder telescopes under construction, *e.g.* LOFAR, ASKAP, MeerKAT, MWA.
- New modelling and imaging techniques required to ensure the next-generation of interferometric telescopes reach their full potential.

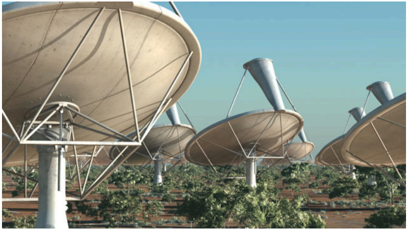
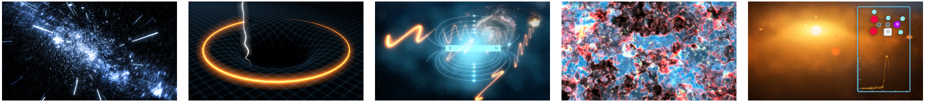


Figure: Artist impression of SKA dishes. [Credit: SKA Organisation]



(a) Dark-energy (b) GR (c) Cosmic magnetism (d) **Epoch of reionization** (e) Exoplanets

Figure: SKA science goals. [Credit: SKA Organisation]

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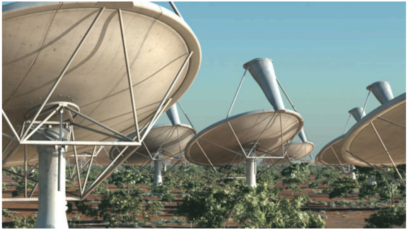

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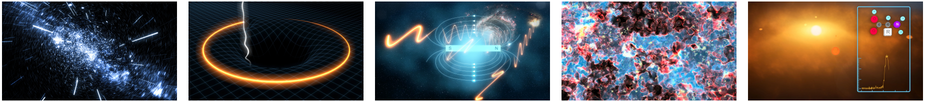

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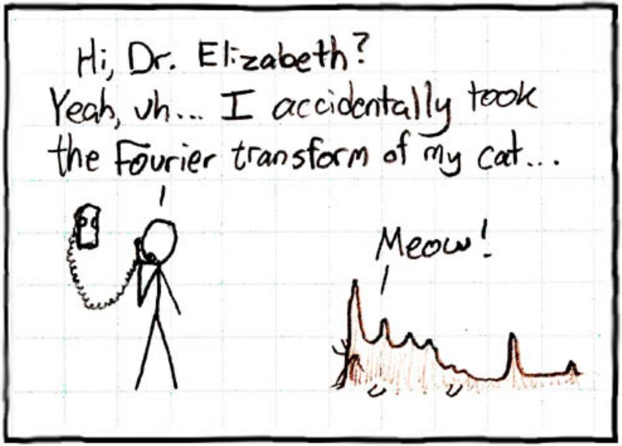
Fourier imaging



[Credit: xkcd]

Radio interferometry

- The complex visibility measured by an interferometer is given by

$$y(\boldsymbol{u}, w) = \int_{D^2} A(\boldsymbol{l})\, x(\boldsymbol{l})\, C(\|\boldsymbol{l}\|_2)\, \mathrm{e}^{-\mathrm{i}2\pi \boldsymbol{u}\cdot\boldsymbol{l}}\, \frac{\mathrm{d}^2\boldsymbol{l}}{n(\boldsymbol{l})},$$

visibilities

where the w-modulation $C(\|\boldsymbol{l}\|_2)$ is given by

$$C(\|\boldsymbol{l}\|_2) \equiv \mathrm{e}^{\mathrm{i}2\pi w\left(1-\sqrt{1-\|\boldsymbol{l}\|^2}\right)}.$$

w-modulation

- Various assumptions are often made regarding the size of the field-of-view (FoV):
  - Small-field with $\|\boldsymbol{l}\|^2 w \ll 1$ ⇒ $C(\|\boldsymbol{l}\|_2) \simeq 1$
  - Small-field with $\|\boldsymbol{l}\|^4 w \ll 1$ ⇒ $C(\|\boldsymbol{l}\|_2) \simeq \mathrm{e}^{\mathrm{i}\pi w\|\boldsymbol{l}\|^2}$
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Radio interferometric inverse problem

- Consider the ill-posed inverse problem of radio interferometric imaging:

$$\boldsymbol{y} = \Phi \boldsymbol{x} + \boldsymbol{n} ,$$

where $\boldsymbol{y}$ are the measured visibilities, Φ is the linear measurement operator, $\boldsymbol{x}$ is the underlying image and $\boldsymbol{n}$ is instrumental noise.

- Measurement operator $\Phi = \mathbf{M}\,\mathbf{F}\,\mathbf{C}\,\mathbf{A}$ may incorporate:
  - primary beam $\mathbf{A}$ of the telescope;
  - w-modulation modulation $\mathbf{C}$;
  - Fourier transform $\mathbf{F}$;
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Interferometric imaging: recover an image from noisy and incomplete Fourier measurements.

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Compressive sensing

"Nothing short of revolutionary."

– National Science Foundation

- Developed by Emmanuel Candes and David Donoho (and others).

(a) Emmanuel Candes

(b) David Donoho

Compressive sensing

- Next evolution of wavelet analysis → wavelets are a key ingredient.
- The mystery of JPEG compression (discrete cosine transform; wavelet transform).
- Move compression to the acquisition stage → compressive sensing.
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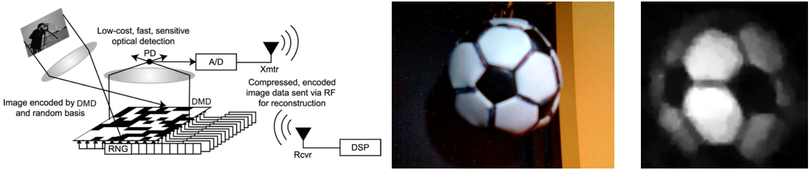

(a) Architecture (b) Scene (c) Recov. (20% meas.)

Figure: Single pixel camera

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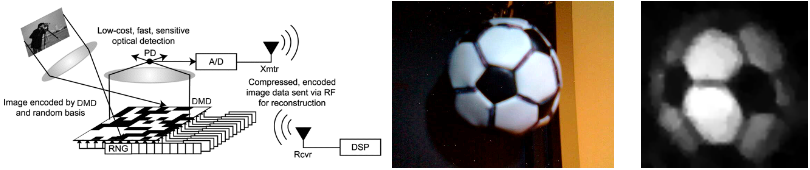

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An introduction to compressive sensing

Operator description

- Linear operator (linear algebra) representation of signal decomposition:

$$x(t) = \sum_i \alpha_i \Psi_i(t) \quad \rightarrow \quad \boldsymbol{x} = \sum_i \Psi_i \alpha_i = \begin{pmatrix} | \\ \Psi_0 \\ | \end{pmatrix} \alpha_0 + \begin{pmatrix} | \\ \Psi_1 \\ | \end{pmatrix} \alpha_1 + \cdots \quad \rightarrow \quad \boxed{\boldsymbol{x} = \Psi \boldsymbol{\alpha}}$$

- Linear operator (linear algebra) representation of measurement:

$$y_i = \langle x, \Phi_j \rangle \quad \rightarrow \quad \boldsymbol{y} = \begin{pmatrix} - \Phi_0 - \\ - \Phi_1 - \\ \vdots \end{pmatrix} \boldsymbol{x} \quad \rightarrow \quad \boxed{\boldsymbol{y} = \Phi \boldsymbol{x}}$$

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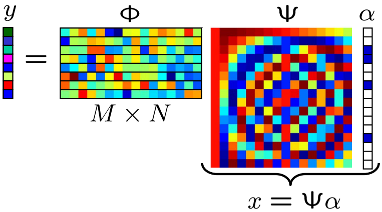

An introduction to compressive sensing

Promoting sparsity via ℓ_1 minimisation

- Ill-posed inverse problem:

$$\boxed{\boldsymbol{y} = \Phi \boldsymbol{x} + \boldsymbol{n} = \Phi \Psi \boldsymbol{\alpha} + \boldsymbol{n}}.$$

- Recall norms given by:

$$\|\alpha\|_0 = \text{no. non-zero elements} \qquad \|\alpha\|_1 = \sum_i |\alpha_i| \qquad \|\alpha\|_2 = \Big(\sum_i |\alpha_i|^2\Big)^{1/2}$$

- Solve by imposing a regularising prior that the signal to be recovered is sparse in Ψ, *i.e.* solve the following ℓ_0 optimisation problem:

$$\boxed{\boldsymbol{\alpha}^\star = \arg\min_{\boldsymbol{\alpha}} \|\alpha\|_0 \text{ such that } \|\boldsymbol{y} - \Phi\Psi\boldsymbol{\alpha}\|_2 \leq \epsilon},$$

where the signal is synthesising by $\boldsymbol{x}^\star = \Psi \boldsymbol{\alpha}^\star$.

- Solving this problem is difficult (combinatorial).
- Instead, solve the ℓ_1 optimisation problem (convex):

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Promoting sparsity via ℓ_1 minimisation

- Solutions of the ℓ_0 and ℓ_1 problems are often the same.
- Restricted isometry property (RIP):

$$(1-\delta_K)\|\boldsymbol{\alpha}\|_2^2 \leq \|\Theta\boldsymbol{\alpha}\|_2^2 \leq (1+\delta_K)\|\boldsymbol{\alpha}\|_2^2 \, ,$$

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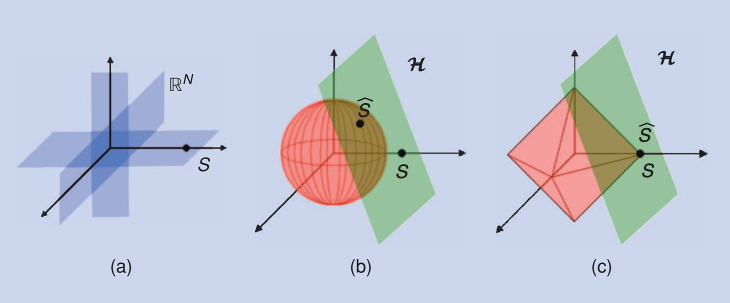

Figure: Geometry of (a) ℓ_0 (b) ℓ_2 and (c) ℓ_1 problems. [Credit: Baraniuk (2007)]

An introduction to compressive sensing

Coherence

- In the absence of noise, compressed sensing is exact!
- Number of measurements required to achieve exact reconstruction is given by

$$M \geq c\mu^2 K \log N ,$$

where K is the sparsity and N the dimensionality.

- The coherence between the measurement and sparsity basis is given by

$$\mu = \sqrt{N} \max_{i,j} |\langle \Psi_i, \Phi_j \rangle| .$$

- Robust to noise.

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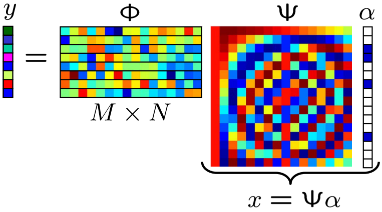

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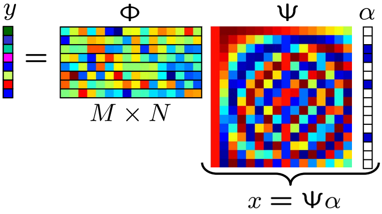

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Analysis vs synthesis

- Many new developments (*e.g.* analysis vs synthesis, cosparsity, structured sparsity).
- Synthesis-based framework:

$$\alpha^\star = \arg\min_{\alpha} \|\alpha\|_1 \ \text{ such that } \ \|y - \Phi\Psi\alpha\|_2 \leq \epsilon \,.$$

where we synthesise the signal from its recovered wavelet coefficients by $x^\star = \Psi\alpha^\star$.

- Analysis-based framework:

$$x^\star = \arg\min_{x} \|\Psi^{\mathrm{T}} x\|_1 \ \text{ such that } \ \|y - \Phi x\|_2 \leq \epsilon \,,$$

where the signal $x^\star$ is recovered directly.

- Concatenating dictionaries (Rauhut *et al.* 2008) and sparsity averaging (Carrillo, McEwen & Wiaux 2013)

$$\Psi = [\Psi_1, \Psi_2, \cdots, \Psi_q] \,.$$

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Interferometric imaging with compressed sensing

- Solve the interferometric imaging problem

$$\boldsymbol{y} = \Phi \boldsymbol{x} + \boldsymbol{n} \quad \text{with} \quad \Phi = \mathbf{M}\,\mathbf{F}\,\mathbf{C}\,\mathbf{A} \,,$$

by applying a prior on sparsity of the signal in a sparsifying dictionary Ψ.

- Basis pursuit (BP) denoising problem

$$\boldsymbol{\alpha}^\star = \underset{\boldsymbol{\alpha}}{\arg\min} \|\boldsymbol{\alpha}\|_1 \text{ such that } \|\boldsymbol{y} - \Phi\Psi\boldsymbol{\alpha}\|_2 \leq \epsilon \,,$$

where the image is synthesised by $\boldsymbol{x}^\star = \Psi\boldsymbol{\alpha}^\star$.

- Total Variation (TV) denoising problem

$$\boldsymbol{x}^\star = \underset{\boldsymbol{x}}{\arg\min} \|\boldsymbol{x}\|_{\text{TV}} \text{ such that } \|\boldsymbol{y} - \Phi\boldsymbol{x}\|_2 \leq \epsilon \,.$$

- Various choices for sparsifying dictionary Ψ, *e.g.* Dirac basis, Daubechies wavelets.

Interferometric imaging with compressed sensing

- Solve the interferometric imaging problem

$$\boxed{\boldsymbol{y} = \Phi \boldsymbol{x} + \boldsymbol{n} \quad \text{with} \quad \Phi = \mathbf{M}\,\mathbf{F}\,\mathbf{C}\,\mathbf{A}} \,,$$

  by applying a prior on sparsity of the signal in a sparsifying dictionary Ψ.

- Basis pursuit (BP) denoising problem

$$\boldsymbol{\alpha}^\star = \underset{\boldsymbol{\alpha}}{\arg\min} \|\alpha\|_1 \;\text{ such that }\; \|\boldsymbol{y} - \Phi\Psi\boldsymbol{\alpha}\|_2 \leq \epsilon \,,$$

  where the image is synthesised by $\boldsymbol{x}^\star = \Psi\boldsymbol{\alpha}^\star$.

- Total Variation (TV) denoising problem

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SARA for radio interferometric imaging
Algorithm

- Sparsity averaging reweighted analysis (SARA) for RI imaging (Carrillo, McEwen & Wiaux 2012)
- Consider a dictionary composed of a concatenation of orthonormal bases, i.e.

$$\Psi = \frac{1}{\sqrt{q}}[\Psi_1, \Psi_2, \ldots, \Psi_q],$$

  thus $\Psi \in \mathbb{R}^{N \times D}$ with $D = qN$.
- We consider the following bases: Dirac (*i.e.* pixel basis); Haar wavelets (promotes gradient sparsity); Daubechies wavelet bases two to eight.
  $\Rightarrow$ concatenation of 9 bases
- Promote average sparsity by solving the reweighted ℓ_1 analysis problem:

$$\min_{\bar{\boldsymbol{x}} \in \mathbb{R}^N} \|W \Psi^T \bar{\boldsymbol{x}}\|_1 \quad \text{subject to} \quad \|\boldsymbol{y} - \Phi \bar{\boldsymbol{x}}\|_2 \leq \epsilon \quad \text{and} \quad \bar{\boldsymbol{x}} \geq 0\,,$$

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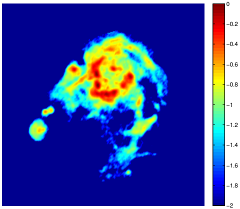
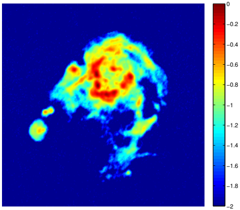
SARA for radio interferometric imaging

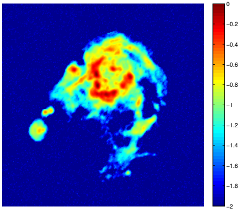
Results on simulations



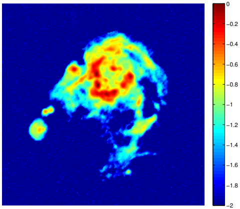
(a) Original



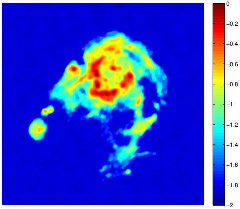
(b) BP (SNR=32.82 dB)



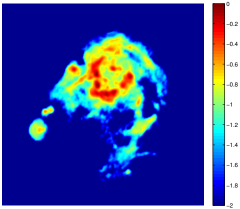
(c) IUWT (SNR=32.12 dB)



(d) BPDb8 (SNR=33.70 dB)



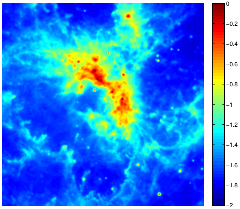
(e) TV (SNR=33.89 dB)



(f) SARA (SNR=38.43 dB)

SARA for radio interferometric imaging

Results on simulations



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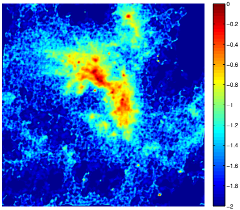
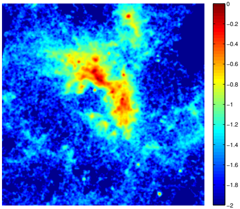
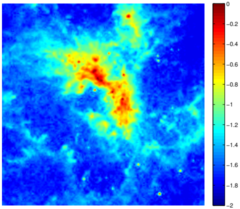


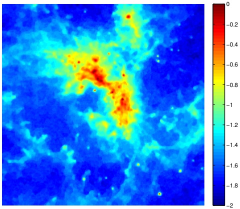
(b) BP (SNR=16.67 dB)



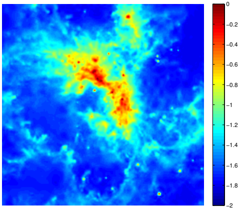
(c) IUWT (SNR=17.87 dB)



(d) BPDb8 (SNR=24.53 dB)



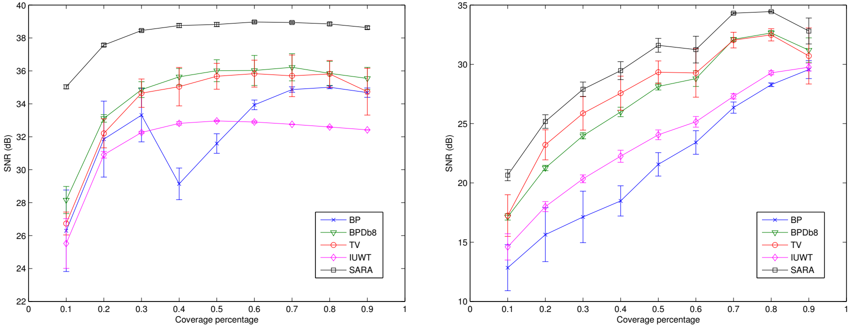
(e) TV (SNR=26.47 dB)



(f) SARA (SNR=29.08 dB)

SARA for radio interferometric imaging

Results on simulations



(a) M31 (b) 30Dor

Figure: Reconstruction fidelity vs visibility coverage.

Outline

1. Radio Interferometry (RI)
2. Compressive Sensing (CS)
3. Radio Interferometric Imaging with Compressive Sensing (RI+CS)
4. Spread Spectrum
5. Continuous Visibilities
6. Outlook

Review of the spread spectrum effect

- Wide field $\rightarrow$ w-modulation $\rightarrow$ spread spectrum effect first considered by Wiaux *et al.* (2009b).
- The w-modulation operator $\mathbf{C}$ has elements defined by

$$C(l,m) \equiv e^{i2\pi w\left(1-\sqrt{1-l^2-m^2}\right)} \simeq e^{i\pi w \|\boldsymbol{l}\|^2} \quad \text{for} \quad \|\boldsymbol{l}\|^4\, w \ll 1 ,$$

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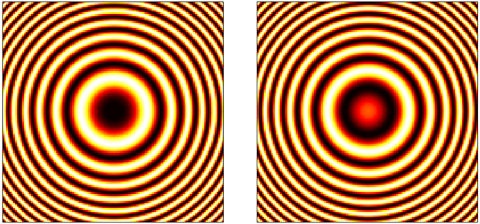

(a) Real part (b) Imaginary part

Figure: Chirp modulation.

Review of the spread spectrum effect

- For the (essentially) Fourier measurements of interferometric telescopes the coherence is the maximum modulus of the Fourier coefficients of atoms of the sparsifying dictionary.
- w-modulation spreads the spectrum of the atoms of the sparsifying dictionary.
- Consequently, spreading the spectrum increases the incoherence between the sensing and sparsity bases, thus improving reconstruction fidelity.
- Improved reconstruction fidelity of the spread spectrum effect demonstrated with simulations by Wiaux *et al.* (2009b).
- However, previous analysis was restricted to constant w for simplicity.
- Examined the spread spectrum effect for varying w.
- Work of Laura Wolz in collaboration with McEwen, Abdalla, Carrillo and Wiaux (see Wolz *et al.* 2013).

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Spread spectrum effect for varying w

w-projection

- Apply the w-projection algorithm (Cornwell *et al.* 2008) to shift the chirp modulation through the Fourier transform:

$$\boxed{\Phi = \mathbf{M}\,\mathbf{F}\,\mathbf{C}\,\mathbf{A} \quad \Rightarrow \quad \Phi = \hat{\mathbf{C}}\,\mathbf{F}\,\mathbf{A}} \; .$$

- Consider different w for each (u, v) and threshold each Fourier transformed chirp (each row of $\hat{\mathbf{C}}$) to approximate $\hat{\mathbf{C}}$ accurately by a sparse matrix.
- Retain E% of the energy content of the w-modulation for each visibility measurement (typically $E = 75\%$).
- Support of w-modulation in Fourier space determined dynamically.

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Approximation of w-modulation kernel

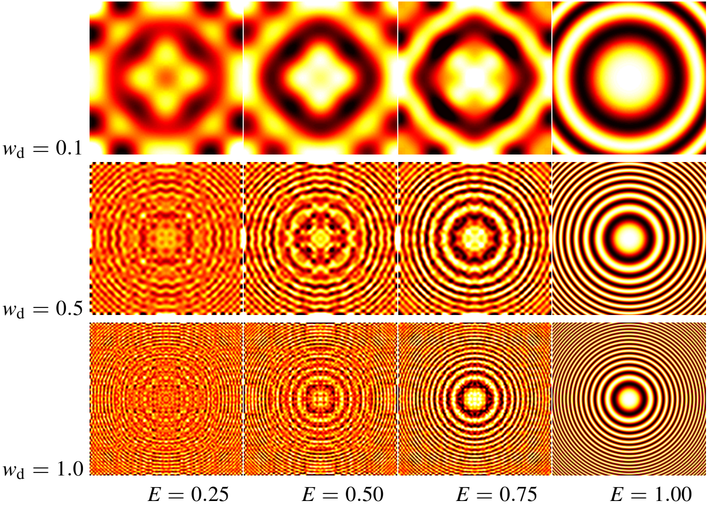

Figure: w-modulation kernel.

Spread spectrum effect for varying w

Impact of approximation of w-modulation kernel

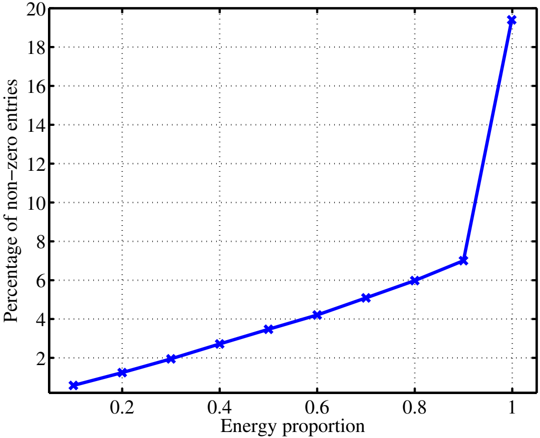

Figure: Percentage of non-zero entries as a function of preserved energy proportion.

Spread spectrum effect for varying w

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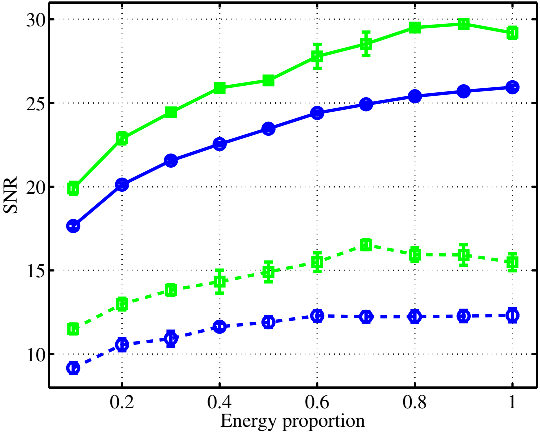
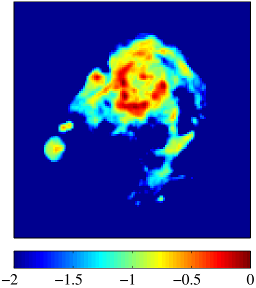


Figure: Reconstruction quality of M31 (green lines marked with squares) and 30Dor (blue lines marked with circles) as a function of preserved energy proportion for visibility coverages 10% (dashed) and 50% (solid).

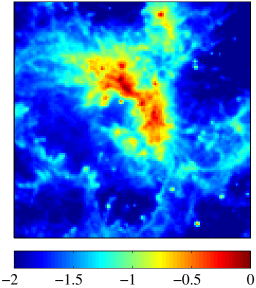
Spread spectrum effect for varying w

Results on simulations

- Perform simulations to assess the effectiveness of the spread spectrum effect in the presence of varying w.
- Consider idealised simulations with uniformly random visibility sampling.



(a) M31

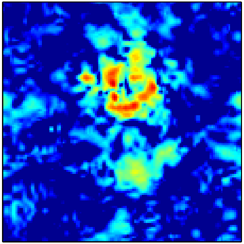


(b) 30Dor

Figure: Ground truth images in logarithmic scale.

Spread spectrum effect for varying w

Results on simulations

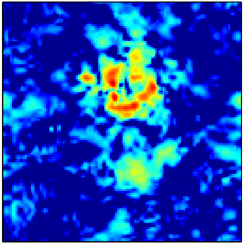


(a) $w_\mathrm{d} = 0 \rightarrow$ SNR$= 5$dB

Figure: Reconstructed images of M31 for 10% coverage.

Spread spectrum effect for varying w

Results on simulations



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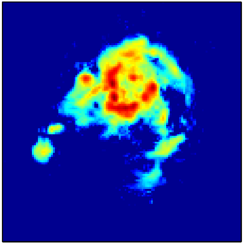

(c) $w_{\mathrm{d}} = 1 \rightarrow$ SNR$=$ 19dB

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Spread spectrum effect for varying w

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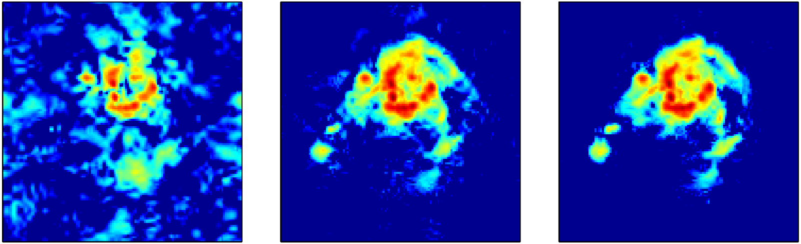

(a) $w_{\rm d} = 0 \rightarrow$ SNR$=$ 5dB (b) $w_{\rm d} \sim \mathcal{U}(0, 1) \rightarrow$ SNR$=$ 16dB (c) $w_{\rm d} = 1 \rightarrow$ SNR$=$ 19dB

Figure: Reconstructed images of M31 for 10% coverage.

Spread spectrum effect for varying w

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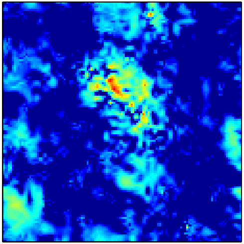
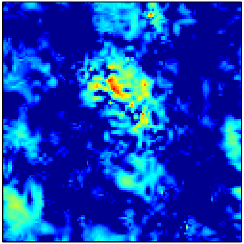


(a) $w_\mathrm{d} = 0 \rightarrow$ SNR$=$ 2dB

Figure: Reconstructed images of 30Dor for 10% coverage.

Spread spectrum effect for varying w

Results on simulations



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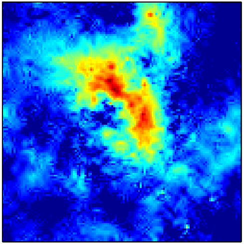

(c) $w_d = 1 \rightarrow$ SNR$=$ 15dB

Figure: Reconstructed images of 30Dor for 10% coverage.

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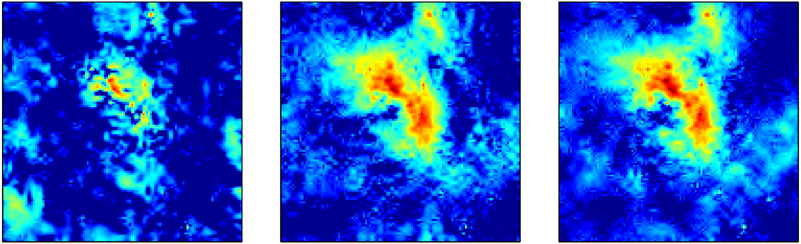

(a) $w_{\rm d} = 0 \rightarrow$ SNR$=$ 2dB (b) $w_{\rm d} \sim \mathcal{U}(0, 1) \rightarrow$ SNR$=$ 12dB (c) $w_{\rm d} = 1 \rightarrow$ SNR$=$ 15dB

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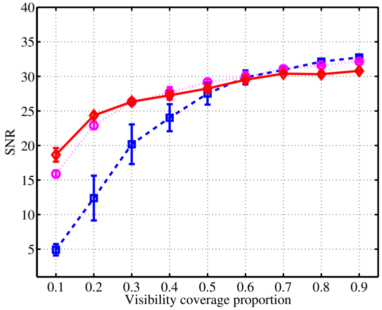


(a) Daubechies 8 (Db8) wavelets

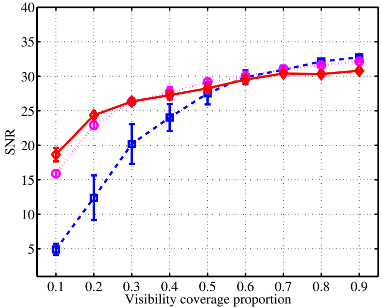
Figure: Reconstruction fidelity for M31.

Improvement in reconstruction fidelity due to the spread spectrum effect for varying w is almost as large as the case of constant maximum w!

- As expected, for the case where coherence is already optimal, there is little improvement.

Spread spectrum effect for varying w

Results on simulations



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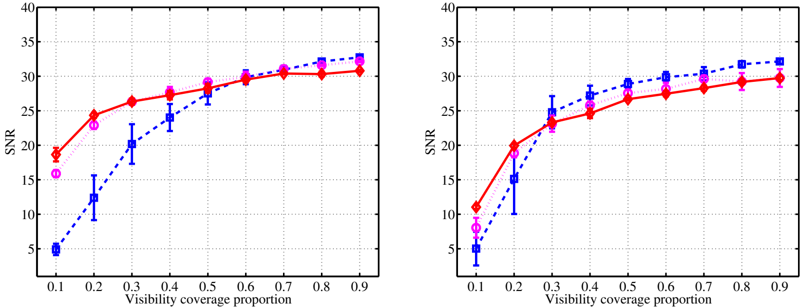

(a) Daubechies 8 (Db8) wavelets (b) Dirac basis

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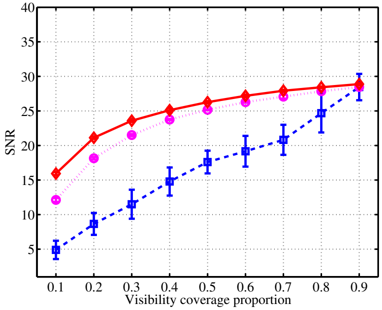

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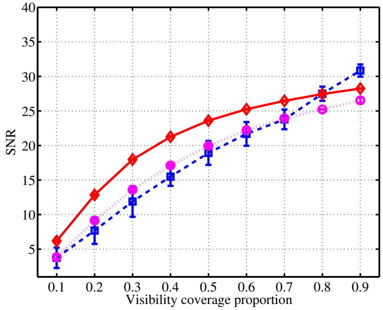

(b) Dirac basis

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Outline

1 Radio Interferometry (RI)

2 Compressive Sensing (CS)

3 Radio Interferometric Imaging with Compressive Sensing (RI+CS)

4 Spread Spectrum

5 Continuous Visibilities

6 Outlook

Supporting continuous visibilities

Algorithm

- Ideally we would like to model the continuous Fourier transform operator

$$\Phi = \mathbf{F}^{c} \, .$$

- But this is impractically slow!
- Incorporated gridding into our CS interferometric imaging framework.
- Work of Rafael Carrillo, in collaboration with Wiaux and McEwen (see Carrillo, McEwen, Wiaux 2013).
- Model with measurement operator

$$\Phi = \mathbf{G}\,\mathbf{F}\,\mathbf{D}\,\mathbf{Z} \, ,$$

where we incorporate:

  - convolutional gridding operator $\mathbf{G}$;
  - fast Fourier transform $\mathbf{F}$;
  - normalisation operator $\mathbf{D}$ to undo the convolution gridding;
  - zero-padding operator $\mathbf{Z}$ to upsample the discrete visibility space.

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  - normalisation operator $\mathbf{D}$ to undo the convolution gridding;
  - zero-padding operator $\mathbf{Z}$ to upsample the discrete visibility space.

Supporting continuous visibilities
Algorithm

- Ideally we would like to model the continuous Fourier transform operator

$$\Phi = \mathbf{F}^{\mathrm{c}} \, .$$

- But this is impractically slow!
- Incorporated gridding into our CS interferometric imaging framework.
- Work of Rafael Carrillo, in collaboration with Wiaux and McEwen (see Carrillo, McEwen, Wiaux 2013).
- Model with measurement operator

$$\Phi = \mathbf{G}\,\mathbf{F}\,\mathbf{D}\,\mathbf{Z} \, ,$$

where we incorporate:

  - convolutional gridding operator $\mathbf{G}$;
  - fast Fourier transform $\mathbf{F}$;
  - normalisation operator $\mathbf{D}$ to undo the convolution gridding;
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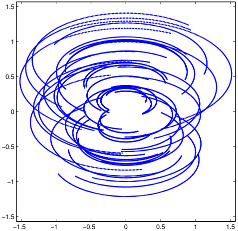
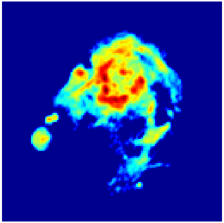
Supporting continuous visibilities

Results on simulations



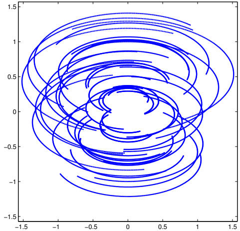
(a) Coverage



(b) M31 (ground truth)

Supporting continuous visibilities

Results on simulations



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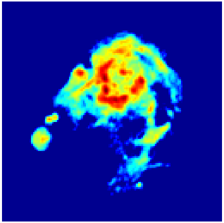

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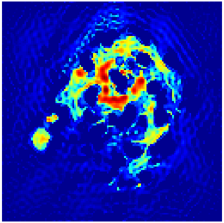
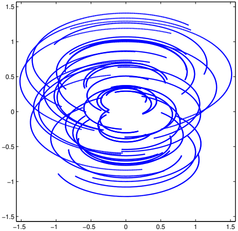


(c) Dirac basis $\rightarrow$ SNR$=8.2$dB

Supporting continuous visibilities

Results on simulations



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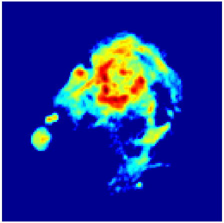

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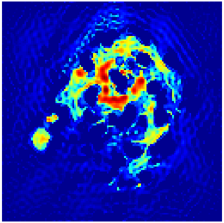

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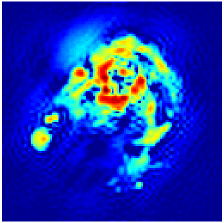
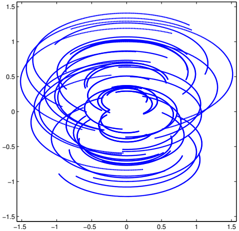


(d) Db8 wavelets → SNR= 11.1dB

Supporting continuous visibilities

Results on simulations



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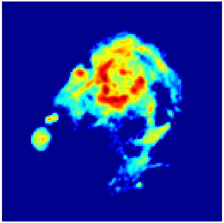

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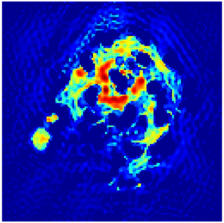

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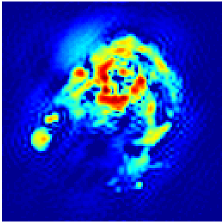

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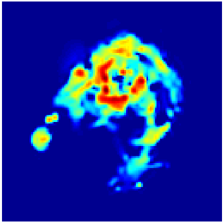

(e) SARA $\rightarrow$ SNR$=13.4$dB

Outline

1. Radio Interferometry (RI)
2. Compressive Sensing (CS)
3. Radio Interferometric Imaging with Compressive Sensing (RI+CS)
4. Spread Spectrum
5. Continuous Visibilities
6. Outlook

Outlook

- Effectiveness of compressive sensing for radio interferometric imaging demonstrated (Wiaux *et al.* 2009a, Wiaux *et al.*2009b, Wiaux *et al.* 2009c, McEwen & Wiaux 2011, Carrillo *et al.* 2012).
- Important to take these methods to the realistic setting so that their advantages can be realised on observations made by real radio interferometric telescopes.

- Taken first steps toward more realistic setting.
- Wide fields: studied the spread spectrum effect for varying w (Wolz *et al.* 2013).
- Continuous visibilities: incorporated gridding operator (Carrillo *et al.* 2013).

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- Includes state-of-the-art convex optimisation algorithms that support parallelisation.
- Plan to perform more extensive comparisons with traditional techniques, such as CLEAN, MS-CLEAN and MEM.

Apply to observations made by real interferometric telescopes.

PURIFY code http://basp-group.github.io/purify/

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Carrillo, McEwen, Wiaux

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