Towards realistic radio interferometric imaging with compressive sensing

Jason McEwen

http://www.jasonmcewen.org/

Department of Physics and Astronomy, University College London (UCL)

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Outline

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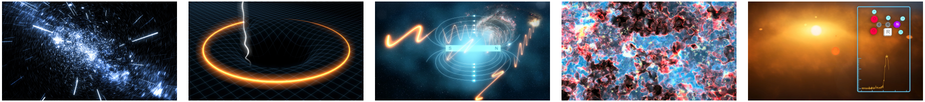
Next-generation of radio interferometry rapidly approaching

- Square Kilometre Array (SKA) first observations planned for 2019.
- Many other pathfinder telescopes under construction, *e.g.* LOFAR, ASKAP, MeerKAT, MWA.
- New modelling and imaging techniques required to ensure the next-generation of interferometric telescopes reach their full potential.

Figure: Artist impression of SKA dishes. [Credit: SKA Organisation]



(a) Dark-energy (b) GR (c) Cosmic magnetism (d) **Epoch of reionization** (e) Exoplanets

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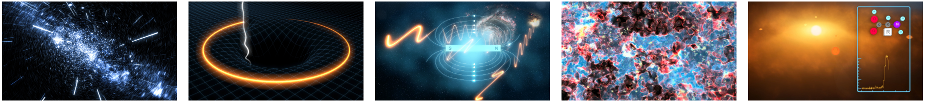

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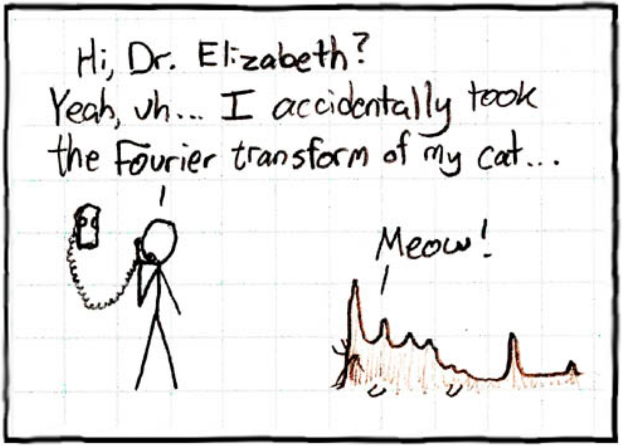
Fourier imaging



[Credit: xkcd]

Radio interferometry

- The complex visibility measured by an interferometer is given by

$$y(\boldsymbol{u}, w) = \int_{D^2} A(\boldsymbol{l})\, x_{\rm p}(\boldsymbol{l})\, {\rm e}^{-{\rm i}2\pi[\boldsymbol{u}\cdot\boldsymbol{l} + w\,(n(\boldsymbol{l})-1)]}\, \frac{{\rm d}^2\boldsymbol{l}}{n(\boldsymbol{l})}$$
$$= \int_{D^2} A(\boldsymbol{l})\, x_{\rm p}(\boldsymbol{l})\, C(\|\boldsymbol{l}\|_2)\, {\rm e}^{-{\rm i}2\pi\boldsymbol{u}\cdot\boldsymbol{l}}\, \frac{{\rm d}^2\boldsymbol{l}}{n(\boldsymbol{l})} ,$$

where $\boldsymbol{l} = (l, m)$, $\|\boldsymbol{l}\|^2 + n^2(\boldsymbol{l}) = 1$ and the w-component $C(\|\boldsymbol{l}\|_2)$ is given by

$$C(\|\boldsymbol{l}\|_2) \equiv {\rm e}^{{\rm i}2\pi w\left(1-\sqrt{1-\|\boldsymbol{l}\|^2}\right)} .$$

- Various assumptions are often made regarding the size of the field-of-view (FoV):
  - Small-field with $\|\boldsymbol{l}\|^2\, w \ll 1 \quad \Rightarrow \quad C(\|\boldsymbol{l}\|_2) \simeq 1$
  - Small-field with $\|\boldsymbol{l}\|^4\, w \ll 1 \quad \Rightarrow \quad C(\|\boldsymbol{l}\|_2) \simeq {\rm e}^{{\rm i}\pi w\|\boldsymbol{l}\|^2}$
  - Wide-field $\quad \Rightarrow \quad C(\|\boldsymbol{l}\|_2) = {\rm e}^{{\rm i}2\pi w\left(1-\sqrt{1-\|\boldsymbol{l}\|^2}\right)}$

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Radio interferometric inverse problem

- Consider the ill-posed inverse problem of radio interferometric imaging:

$$\boxed{\boldsymbol{y} = \Phi \boldsymbol{x} + \boldsymbol{n}} \, ,$$

where $\boldsymbol{y}$ are the measured visibilities, Φ is the linear measurement operator, $\boldsymbol{x}$ is the underlying image and $\boldsymbol{n}$ is instrumental noise.

- Measurement operator $\boxed{\Phi = \mathbf{M}\,\mathbf{F}\,\mathbf{C}\,\mathbf{A}}$ may incorporate:
  - primary beam $\mathbf{A}$ of the telescope;
  - w-component modulation $\mathbf{C}$ (responsible for the spread spectrum phenomenon);
  - Fourier transform $\mathbf{F}$;
  - masking $\mathbf{M}$ which encodes the incomplete measurements taken by the interferometer.

- Interferometric imaging: recover an image from noisy and incomplete Fourier measurements.

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(a) Emmanuel Candes

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Compressive sensing

- Next evolution of wavelet analysis $\rightarrow$ wavelets are a key ingredient.
- The mystery of JPEG compression (discrete cosine transform; wavelet transform).
- Move compression to the acquisition stage $\rightarrow$ compressive sensing.
- Acquisition versus imaging.

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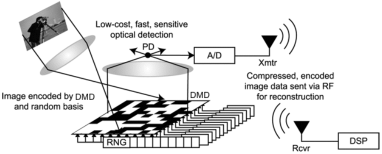

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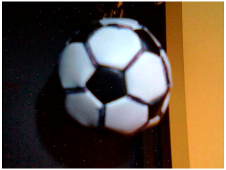

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(c) Recov. (20% meas.)

Figure: Single pixel camera

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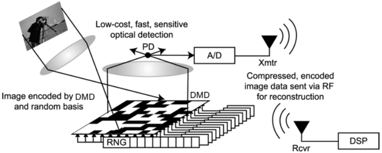

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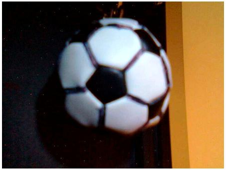

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An introduction to compressive sensing

- Linear operator (linear algebra) representation of signal decomposition:

$$x(t) = \sum_i \alpha_i \Psi_i(t) \quad \rightarrow \quad \boldsymbol{x} = \sum_i \Psi_i \alpha_i = \begin{pmatrix} | \\ \Psi_0 \\ | \end{pmatrix} \alpha_0 + \begin{pmatrix} | \\ \Psi_1 \\ | \end{pmatrix} \alpha_1 + \cdots \quad \rightarrow \quad \boxed{\boldsymbol{x} = \Psi \boldsymbol{\alpha}}$$

- Linear operator (linear algebra) representation of measurement:

$$y_i = \langle x, \Phi_j \rangle \quad \rightarrow \quad \boldsymbol{y} = \begin{pmatrix} - \Phi_0 - \\ - \Phi_1 - \\ \vdots \end{pmatrix} \boldsymbol{x} \quad \rightarrow \quad \boxed{\boldsymbol{y} = \Phi \boldsymbol{x}}$$

- Putting it together: $\boxed{\boldsymbol{y} = \Phi \boldsymbol{x} = \Phi \Psi \boldsymbol{\alpha}}$

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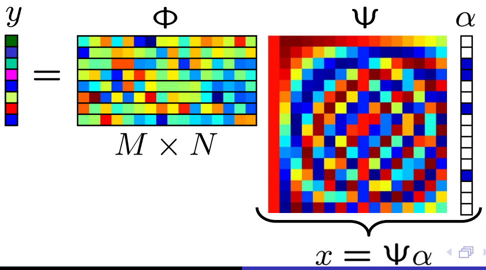

An introduction to compressive sensing

- Ill-posed inverse problem:
$$\boldsymbol{y} = \Phi \boldsymbol{x} + \boldsymbol{n} = \Phi \Psi \boldsymbol{\alpha} + \boldsymbol{n} .$$

- Solve by imposing a regularising prior that the signal to be recovered is sparse in Ψ, *i.e.* solve the following ℓ_0 optimisation problem:
$$\boldsymbol{\alpha}^\star = \underset{\boldsymbol{\alpha}}{\arg\min} \|\alpha\|_0 \text{ such that } \|\boldsymbol{y} - \Phi\Psi\boldsymbol{\alpha}\|_2 \leq \epsilon ,$$
where the signal is synthesising by $\boldsymbol{x}^\star = \Psi \boldsymbol{\alpha}^\star$.

- Recall norms given by:
$$\|\alpha\|_0 = \text{no. non-zero elements} \qquad \|\alpha\|_1 = \sum_i |\alpha_i| \qquad \|\alpha\|_2 = \Big(\sum_i |\alpha_i|^2\Big)^{1/2}$$

- Solving this problem is difficult (combinatorial).

- Instead, solve the ℓ_1 optimisation problem (convex):
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- The solutions of the ℓ_0 and ℓ_1 problems are often the same.
- Restricted isometry property (RIP):

$$(1-\delta_K)\|\boldsymbol{\alpha}\|_2^2 \leq \|\Theta\boldsymbol{\alpha}\|_2^2 \leq (1+\delta_K)\|\boldsymbol{\alpha}\|_2^2 ,$$

for K-sparse $\boldsymbol{\alpha}$, where $\Theta = \Phi\Psi$.

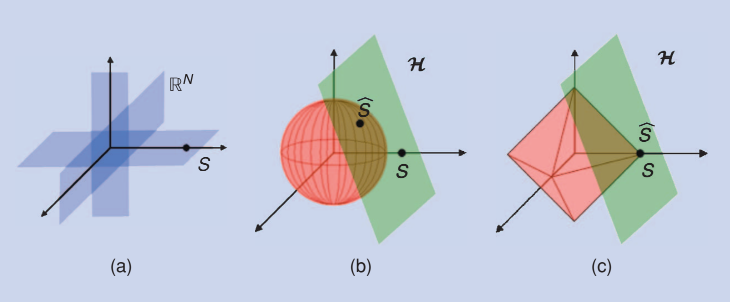

Figure: Geometry of (a) ℓ_0 (b) ℓ_2 and (c) ℓ_1 problems. [Credit: Baraniuk (2007)]

An introduction to compressive sensing

- In the absence of noise, compressed sensing is **exact!**
- Number of measurements required to achieve exact reconstruction is given by

$$M \geq c\mu^2 K \log N \, ,$$

where K is the sparsity and N the dimensionality.

- The coherence between the measurement and sparsity basis is given by

$$\mu = \sqrt{N} \max_{i,j} |\langle \Psi_i, \Phi_j \rangle| \, .$$

- Robust to noise.

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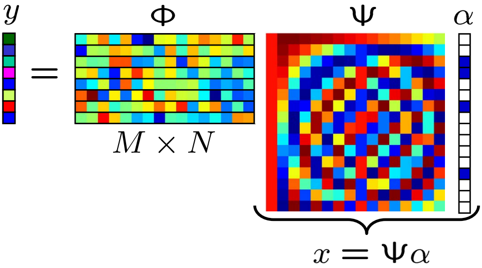

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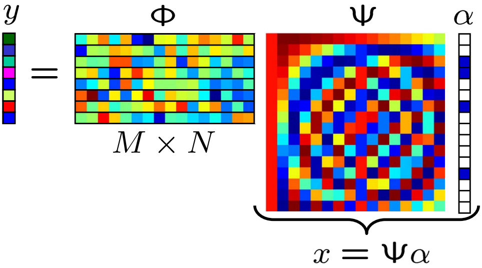

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An introduction to compressive sensing

- Many new developments (*e.g.* analysis vs synthesis, cosparsity, structured sparsity).
- Synthesis-based framework:

$$\alpha^\star = \underset{\alpha}{\arg\min} \|\alpha\|_1 \text{ such that } \|y - \Phi\Psi\alpha\|_2 \leq \epsilon .$$

  where we synthesise the signal from its recovered wavelet coefficients by $x^\star = \Psi\alpha^\star$.
- Analysis-based framework:

$$x^\star = \underset{x}{\arg\min} \|\Psi^{\mathrm{T}}x\|_1 \text{ such that } \|y - \Phi x\|_2 \leq \epsilon ,$$

  where the signal $x^\star$ is recovered directly.
- Concatenating dictionaries (Rauhut *et al.* 2008) and sparsity averaging (Carrillo, JDM & Wiaux 2013)

$$\Psi = [\Psi_1, \Psi_2, \cdots, \Psi_q] .$$

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Interferometric imaging with compressed sensing

- Solve the interferometric imaging problem

$$\boldsymbol{y} = \Phi \boldsymbol{x} + \boldsymbol{n} \quad \text{with} \quad \Phi = \mathbf{M}\,\mathbf{F}\,\mathbf{C}\,\mathbf{A} \,,$$

  by applying a prior on sparsity of the signal in a sparsifying dictionary Ψ.

- Basis pursuit denoising problem

$$\boldsymbol{\alpha}^\star = \underset{\boldsymbol{\alpha}}{\arg\min} \|\alpha\|_1 \ \text{such that} \ \|\boldsymbol{y} - \Phi\Psi\boldsymbol{\alpha}\|_2 \leq \epsilon \,,$$

  where the image is synthesised by $\boldsymbol{x}^\star = \Psi\boldsymbol{\alpha}^\star$.

- Total Variation (TV) denoising problem

$$\boldsymbol{x}^\star = \underset{\boldsymbol{x}}{\arg\min} \|\boldsymbol{x}\|_{\text{TV}} \ \text{such that} \ \|\boldsymbol{y} - \Phi\boldsymbol{x}\|_2 \leq \epsilon \,.$$

- Various choices for sparsifying dictionary Ψ, *e.g.* Dirac basis, Daubechies wavelets.
- Analysis versus synthesis problems, *e.g.* SARA algorithm.
- Recall the potential trade-off between sparsity and coherence.

Interferometric imaging with compressed sensing

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SARA for radio interferometric imaging

- Sparsity averaging reweighted analysis (SARA) for RI imaging (Carrillo, JDM & Wiaux 2012)
- Consider a dictionary composed of a concatenation of orthonormal bases, i.e.

$$\Psi = \frac{1}{\sqrt{q}}[\Psi_1, \Psi_2, \ldots, \Psi_q],$$

thus $\Psi \in \mathbb{R}^{N\times D}$ with $D = qN$.

- We consider the following bases:
  - Dirac, *i.e.* pixel basis
  - Haar wavelets (promotes gradient sparsity)
  - Daubechies wavelet bases two to eight.

$\Rightarrow$ concatenation of 9 bases

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where $W \in \mathbb{R}^{D\times D}$ is a diagonal matrix with positive weights.

- Solve a sequence of reweighted ℓ_1 problems using the solution of the previous problem as the inverse weights $\rightarrow$ approximate the ℓ_0 problem.

SARA for radio interferometric imaging

- Sparsity averaging reweighted analysis (SARA) for RI imaging (Carrillo, JDM & Wiaux 2012)
- Consider a dictionary composed of a concatenation of orthonormal bases, i.e.

$$\Psi = \frac{1}{\sqrt{q}}[\Psi_1, \Psi_2, \ldots, \Psi_q],$$

thus $\Psi \in \mathbb{R}^{N \times D}$ with $D = qN$.

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  - Dirac, *i.e.* pixel basis
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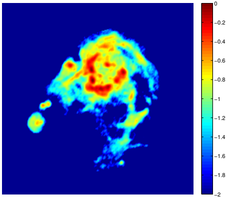
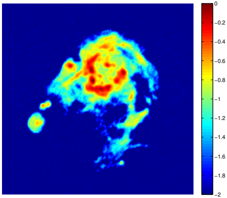
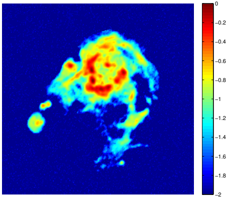
SARA for RI imaging



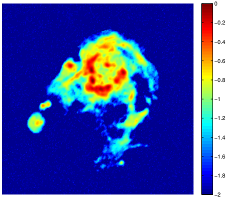
(a) Original



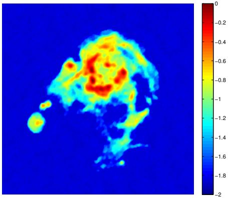
(b) BP (SNR=32.82 dB)



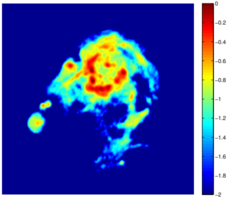
(c) IUWT (SNR=32.12 dB)



(d) BPDb8 (SNR=33.70 dB)



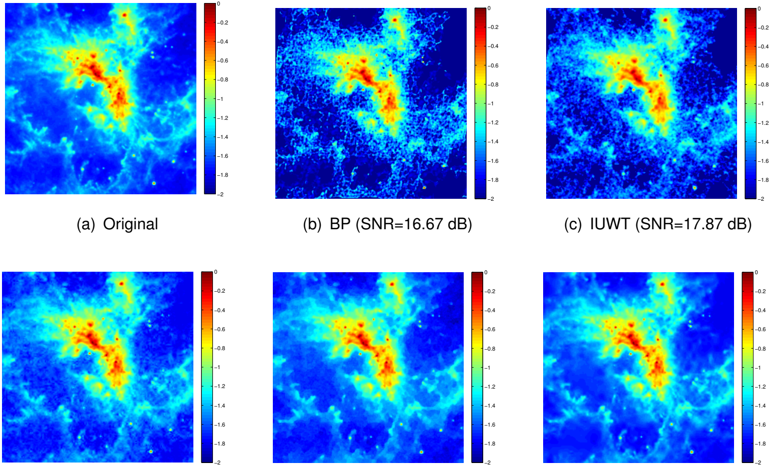
(e) TV (SNR=33.89 dB)



(f) SARA (SNR=38.43 dB)

Figure: Reconstruction example of M31 from 30% of visibilities.

SARA for radio interferometric imaging



(a) Original

(b) BP (SNR=16.67 dB)

(c) IUWT (SNR=17.87 dB)

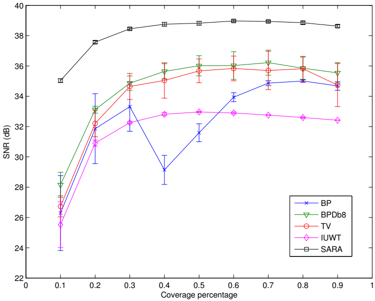
(d) BPDb8 (SNR=24.53 dB)

(e) TV (SNR=26.47 dB)

(f) SARA (SNR=29.08 dB)

Figure: Reconstruction example of 30Dor from 30% of visibilities.

SARA for RI imaging



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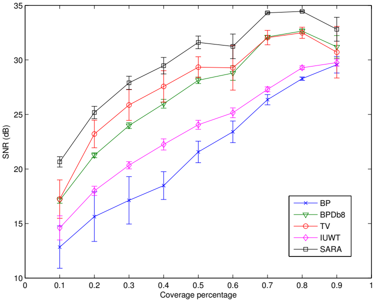

(b) 30Dor

Figure: Reconstruction fidelity vs visibility coverage.

Outline

Review of the spread spectrum phenomenon

- The w-component modulation gives rise to the spread spectrum phenomenon first considered by Wiaux *et al.* (2009b).
- The w-component operator $\mathbf{C}$ has elements defined by

$$C(l, m) \equiv e^{i2\pi w\left(1-\sqrt{1-l^2-m^2}\right)} \simeq e^{i\pi w \|\boldsymbol{l}\|^2} \quad \text{for} \quad \|\boldsymbol{l}\|^4 w \ll 1 ,$$

giving rise to to a linear chirp.

- For the (essentially) Fourier measurements of interferometric telescopes the coherence is the maximum modulus of the Fourier transform of the atoms of the sparsifying dictionary.
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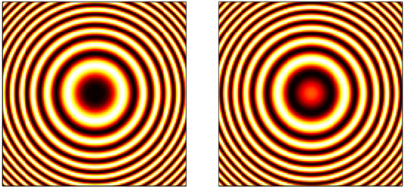

(a) Real part (b) Imaginary part

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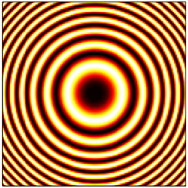
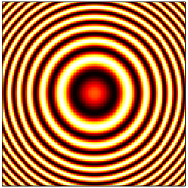


(a) Real part



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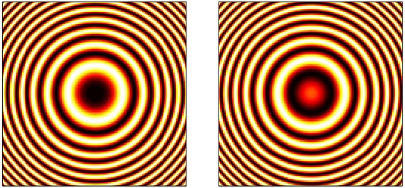

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Spread spectrum phenomenon for varying w

- Improved reconstruction fidelity of the spread spectrum phenomenon demonstrated with simulations by Wiaux *et al.* (2009b).
- However, previous analysis was restricted to fixed w for simplicity.
- Recently, we have examined the spread spectrum phenomenon for varying w.
- Work of Laura Wolz, in collaboration with JDM, Filipe Abdalla, Rafael Carrillo and Yves Wiaux.
- Apply the w-projection algorithm (Cornwell *et al.* 2008) to shift the chirp modulation through the Fourier transform:

$$\boxed{\Phi = \mathbf{M}\,\mathbf{F}\,\mathbf{C}\,\mathbf{A} \quad \Rightarrow \quad \Phi = \mathbf{M}\,\tilde{\mathbf{C}}\,\mathbf{F}\,\mathbf{A}}\,.$$

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Spread spectrum phenomenon for varying w

- Perform simulations to assess the effectiveness of the spread spectrum phenomenon in the presence of varying w.
- Consider idealised simulations with uniformly random visibility sampling.

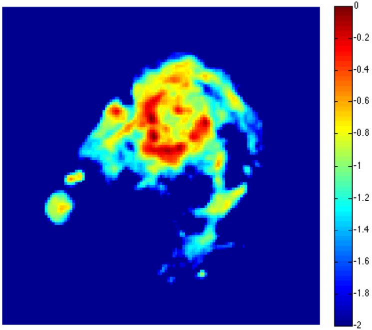
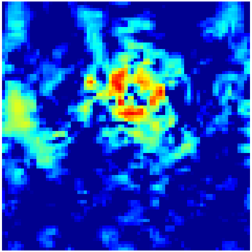


Figure: M31 (ground truth).

Spread spectrum phenomenon for varying w



(a) $w_{\rm d} = 0 \rightarrow$ SNR$= 4.8$dB

Figure: Reconstructed images for 10% coverage.

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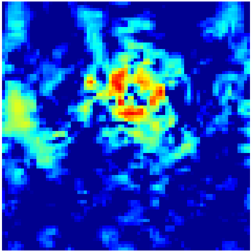

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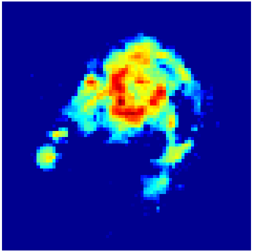

(c) $w_{\mathrm{d}} = 1 \rightarrow$ SNR$= 19.8$dB

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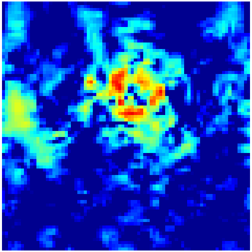

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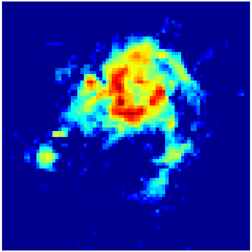
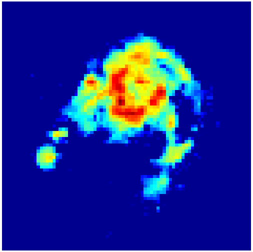


(b) $w_{\mathrm{d}} \sim \mathcal{U}(0, 1) \rightarrow$ SNR$= 16.7$dB



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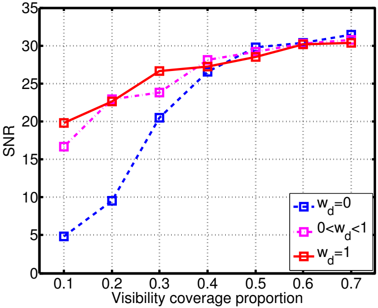


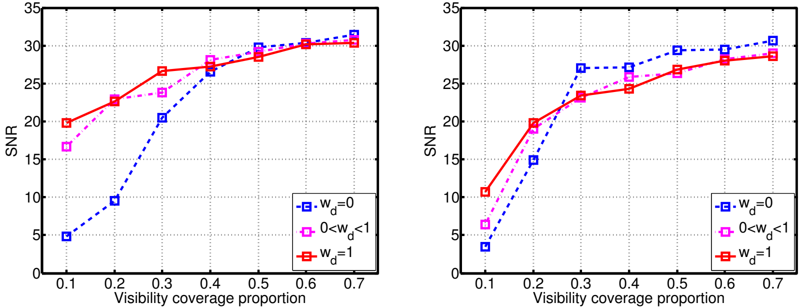
(a) Daubechies 8 (Db8) wavelets

Figure: Reconstruction fidelity.

The improvement in reconstruction fidelity due to the spread spectrum phenomenon for varying w is almost as large as the case of constant maximum w!

- As expected, for the case where coherence is already optimal, there is little improvement.

Spread spectrum phenomenon for varying w



(a) Daubechies 8 (Db8) wavelets (b) Dirac basis

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Outline

Supporting continuous visibilities

- Ideally we would like to model the continuous Fourier transform operator

$$\mathbf{\Phi} = \mathbf{F}^{\mathrm{c}} .$$

- But this is slow!
- We have incorporated gridding into our CS interferometric imaging framework.
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- Model with the measurement operator

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where we incorporate:

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  - normalisation operator $\mathbf{D}$ to undo the convolution gridding (reciprocal of the inverse Fourier transform of the gridding kernel).

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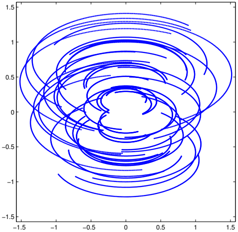
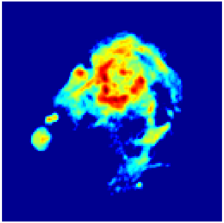
Reconstruction with continuous visibilities



(a) Coverage



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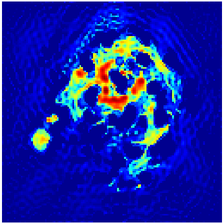
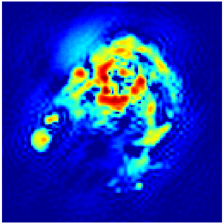
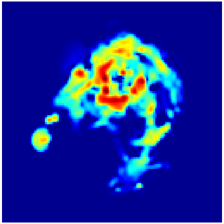


(c) Dirac basis → SNR= 8.2dB



(d) Db8 wavelets → SNR= 11.1dB



(e) SARA → SNR= 13.4dB

Figure: Reconstructed images from continuous visibilities.

Outline

Outlook

- Effectiveness of compressive sensing for radio interferometric imaging demonstrated already (Wiaux *et al.* 2009a, Wiaux *et al.*2009b, Wiaux *et al.* 2009c, JDM & Wiaux 2011, Carrillo *et al.* 2012).
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- Important to take these methods to the realistic setting so that their advantages can be realised on observations made by real radio interferometric telescopes.
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